



# Mathematics Content Framework

## VERSION 1.0 - SY12-13







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## COMMON CORE | CPS

June, 2012

Dear CPS Educators,

We're pleased to share the first version of the CPS **Mathematics Content Framework**. This Framework provides a clear path toward implementation of the Common Core State Standards for Mathematics (CCSS-M), a more rigorous set of expectations to help our students develop the critical analysis and problem-solving skills necessary for college and career success.

The development of this Framework was greatly informed by the Partnership for Assessment of Readiness for College and Careers (PARCC) Model Mathematics Content Framework as well as other mathematics education organizations. The PARCC Framework outlines the critical focus areas and progression of skill development necessary to meet the expectations of the CCSS-M. Teachers from schools across the District, including our Early Adopter pilot schools, collaborated with the Department of Mathematics and Science to build this Framework, and critical feedback was provided by Network teams, as well as local and national mathematics experts. We cannot thank them enough for their thoughtful feedback.

The Framework provides clear expectations for implementation in the form of **Planning Guides** that outline the scope of learning for the 2012-2013 school year. These Planning Guides provide guidance to help teachers build deep conceptual understanding of mathematics, procedural fluency, and critical-thinking and problem-solving skills. The Planning Guides identify sample tasks that are designed to measure the conceptual understanding and mathematical thinking expected of the CCSS-M. Many of these tasks are built from the Mathematics Assessment Resource Service (MARS) tasks. CPS is taking a gradual transition approach to the CCSS-M, and as such it is important to note that the Planning Guides were based on a three-year **Bridge Plan**. The Bridge Plan outlines the transition across all grades to full implementation of all CCSS-M standards by 2014-15.

The Framework also references a Toolset that provides a sample lesson plan template, sample grade-specific tasks, a tool for examining and modifying lessons, and a professional resource list. Included in this document are sample tasks for grade 8 and Algebra I. Complete Toolsets for grades 6-8, as well as Algebra I and Geometry, will be available on our Knowledge Management site: <https://ocs.cps.k12.il.us/sites/IKMC>, and on the Department of Mathematics and Science website at <http://cmsi.cps.k12.il.us/>.

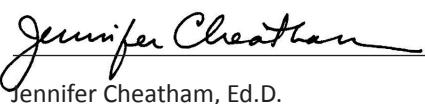
Before diving into this document, it's important to first become familiar with the standards. Earlier this year, every CPS teacher received a personal copy of the standards. Please read through the standards, take note of the learning progressions and the marriage between the standards for mathematical content and practice, and reflect on what shifts will occur in your classroom as a result.

As always, if you have feedback, ideas for resources or have questions about the new standards, do not hesitate to contact us at [commoncore@cps.edu](mailto:commoncore@cps.edu).

We look forward to continuing to work with mathematics educators to further refine our strategy and continue to provide support and resources for implementation. This journey together will help ensure that all students reach a level of achievement that puts them on the path to success in college and career. Thank you for all you do every day for our students.

Sincerely,

  
\_\_\_\_\_  
Jean-Claude Brizard

  
\_\_\_\_\_  
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# Acknowledgements

The work represented in this document and all associated pieces represents a collaborative effort on the part of many. Teachers and Instructional Support Leaders (ISLs) throughout CPS contributed countless hours to share their best thinking and extensive experience in planning, developing, reviewing, and revising these materials. While these materials represent our best understanding about how to address the challenge of implementing the Common Core State Standards for Mathematics in every CPS classroom, we know there is much to be learned. As we begin our transition to meeting this challenge, we welcome feedback about ways to make the materials more useful.

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- University of Illinois at Chicago, Learning Sciences Research Institute

# Overview

The Common Core State Standards for Mathematics (CCSS-M), initiated by the Council of Chief State School Officers (CCSSO) and the National Governors Association (NGA), articulate the skills and understandings that K-12 students must demonstrate in order to be college- and career-ready in mathematics by the end of high school. The CCSS-M are unprecedented in their unified vision of what students are expected to achieve, and the standards are more cohesive and challenging than what has typically existed before. As of spring of 2012, these standards have been adopted by 46 states.

While the Common Core provide the expected results for students' achievement, there is no mandate for how teachers are to instruct. "Teachers are thus free to provide students with whatever tools and knowledge their professional judgment and experience identify as most helpful for meeting the goals set out in the Standards" (CCSS, 2010, p. 4). Therefore, the CPS Department of Mathematics and Science has been collaborating with teachers to develop the **CPS Mathematics Content Framework**—a tool that will guide teachers as they implement the CCSS-M.

Making the transition to these new higher standards will not be easy. All educators, at every level of the system, must work together to create the structures and supports that will help teachers provide rigorous learning experiences in their classrooms every day, experiences that are aligned with the new standards, so that every student is prepared for success after high school.

This document includes:

- An introduction to the Common Core State Standards for Mathematics
- The instructional shifts needed to implement the CCSS-M and how these shifts are evident in the CPS Mathematics Content Framework
  - Focus on critical areas to develop deep conceptual understanding and procedural fluency
  - Integrate the mathematical practice standards throughout instruction
  - Maintain coherence and continuity to link learning within and across grades
- Components of the CPS Mathematics Content Framework
  - CPS Mathematics Bridge Plan
  - CPS Mathematics Planning Guides
  - How Performance Assessments support the CPS Mathematics Content Framework
  - Mathematics Toolsets: Sample Tasks (for each planning guide), Lesson Plan Template, Tool for Examining and Modifying Lessons/Tasks, Sample of Modified Lessons/Tasks, and Recommended Professional Resource List
- A suggested process for designing and implementing a Standards-based curriculum

# An Introduction

## The Common Core State Standards for Mathematics

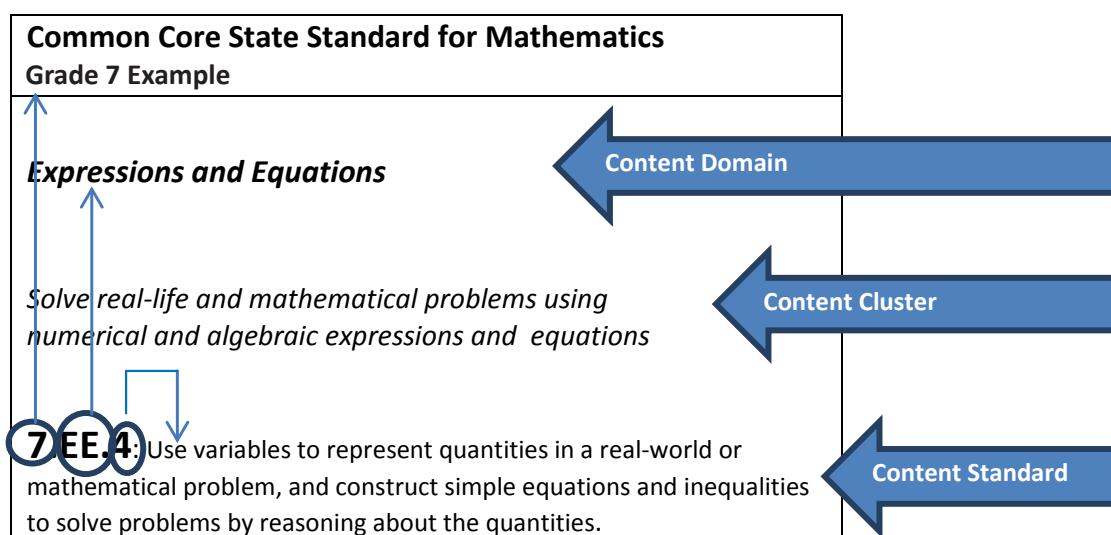
The Common Core State Standards for Mathematics, CCSS-M, define what students should understand and be able to do in mathematics. They set grade-specific standards that follow coherent learning progressions, progressions that reflect what research tells us about how a student's mathematical knowledge, skill, and understanding develop over time. The standards were developed to address the critical need to develop all students so they can succeed in college and careers and be competitive in the global workforce. They raise the bar dramatically for what we expect students to know and be able to do.

### K-8 Content Domains

In K-8, the emphasis of the CCSS-M is all about focus on mastery of the critical skills at each grade. No longer will curricula be "a mile wide but an inch deep." This focus is different from prior standards, and will allow more time to develop fluency in key skills as well as deep conceptual understanding.

The CCSS-M define, by each grade level, what students should understand and be able to do. The content standards are structured in a hierarchy: clusters are groups of related standards; domains are larger groups of related standards.

For example, in a Grade 7 standard, the hierarchy looks like this:



There are eleven content domains across grades K-8. The *Expressions and Equations* domain is highlighted below to illustrate the focus of Year 1, as described below, on page 17.

### Content Domains, K-8

| K                                 | 1 | 2 | 3         | 4 | 5 | 6                                     | 7 | 8         |
|-----------------------------------|---|---|-----------|---|---|---------------------------------------|---|-----------|
| Counting & Cardinality            |   |   |           |   |   |                                       |   |           |
| Operations and Algebraic Thinking |   |   |           |   |   |                                       |   |           |
| Number and Operations in Base 10  |   |   |           |   |   |                                       |   |           |
|                                   |   |   | Fractions |   |   |                                       |   |           |
| Measurement and Data              |   |   |           |   |   |                                       |   |           |
| Geometry                          |   |   |           |   |   |                                       |   |           |
|                                   |   |   |           |   |   | Ratios and Proportional Relationships |   |           |
|                                   |   |   |           |   |   | The Number System                     |   |           |
|                                   |   |   |           |   |   | Statistics and Probability            |   |           |
|                                   |   |   |           |   |   | Expressions and Equations             |   |           |
|                                   |   |   |           |   |   |                                       |   | Functions |
| K                                 | 1 | 2 | 3         | 4 | 5 | 6                                     | 7 | 8         |

The sequencing of skill development represented by these content domains is different than the sequence found in the former Illinois State Learning Standards and in all instructional materials. The work of transitioning to the CCSS-M sequence of skill development for CPS, K-8, is described in the CPS Bridge Plan for Mathematics and in grade-level Planning Guides, described later in this document.

### 9-12 Content Standards

At the high school level, the focus of the CCSS-M is the complex application of what students have learned, K-8. This is captured by the fact that modeling is listed as its own conceptual category. No conceptual category is isolated or addressed by a single high school mathematics course, and every high school course includes content standards from more than one conceptual category. The conceptual categories are:

- Number and Quantity
- Algebra
- Functions
- Modeling
- Geometry
- Statistics and Probability

A student's work in each conceptual category will cross the boundaries of a number of traditional high school mathematics courses. The work of integrating the content standards associated with these conceptual categories across the traditional course structure found in most CPS high schools is addressed in the CPS Mathematics Planning Guides, described later in this document.

### Common Core Standards for Mathematical Practice: K-12

The mathematical practices describe how we expect students to engage with the content and represent varieties of expertise that educators should seek to develop in their students, expertise that students should demonstrate in solving mathematics problems. When compared to the former Illinois state standards, the CCSS-M practices represent a new challenge: the practices need to become an integral part of mathematics instruction and must be incorporated into the lesson along with, not apart from, the content standards.

Teaching and learning at every grade should incorporate all Standards for Mathematical Practice. Mathematics teachers, K-12, should provide in every lesson the opportunity to develop and demonstrate mastery of these practices.

| <b>The CCSS-M Standards for Mathematical Practice</b>               |
|---|
| 1. Make sense of problems and persevere in solving them.            |
| 2. Reason abstractly and quantitatively.                            |
| 3. Construct viable arguments and critique the reasoning of others. |
| 4. Model with mathematics.  |
| 5. Use appropriate tools strategically.                             |
| 6. Attend to precision.   |
| 7. Look for and make use of structure.                              |
| 8. Look for and express regularity in repeated reasoning.           |

Although the Standards for Mathematical Practice are enumerated individually, they are integrated into the content standards. This means that in the transition to CCSS-M described below, the practices are explicitly connected to the content standards, because the practices are not meant to be –in fact cannot be – taught independently.

# CPS Instructional Shifts in Mathematics

The creation of the CPS instructional shifts in mathematics, below, was informed by (1) carefully analyzing the CCSS-M and (2) synthesizing other documents that describe instructional shifts, documents that were created by other states school districts and educational partners. By simplifying and consolidating the shifts, we hope CPS mathematics educators will be able to more easily understand and adopt them.

Teachers are likely to be in different stages in practicing these shifts; however, focusing on them will help us build a common understanding of what is needed in mathematics instruction as we move forward towards full implementation.

| Instructional Shifts in Mathematics   |
|---|
| <b>Focus</b> on critical areas to develop deep conceptual understanding and procedural fluency. |
| <b>Integrate</b> the mathematical practice standards throughout instruction.                    |
| <b>Maintain coherence</b> and continuity to link learning within and across grades.             |

## **Shift 1: Focus on the critical areas to develop deep conceptual understanding and procedural fluency**

### **What does it mean to focus on critical areas to develop deep conceptual understanding and procedural fluency?**

The CCSS-M ask us to teach more deeply to a more focused set of standards. According to Student Achievement Partners, “As a first step in implementing the Common Core State Standards for Mathematics focus strongly where the standards focus.”

In classrooms, this means that teachers understand how the CCSS-M prioritize standards within learning progressions, so that instructional time and energy are focused on critical concepts in a given grade. In this way, students develop strong foundational knowledge and deep conceptual understanding and are able to transfer mathematical skills and understanding across concepts and grades.

As the Standards state, “Conceptual understanding and procedural skill are equally important” (CCSS, 2010, p. 4). In mathematics, students need procedural fluency that requires them to not only know algorithms, but also how and when to use them appropriately. Students need to be able to compute flexibly, accurately and efficiently. This will help them access more complex concepts in later mathematics.

### **Why is this shift so important?**

This shift is important because, as educators, we are often asked to cover a wide range of concepts in each grade level or course. Teaching fewer concepts means educators will have time to be more intentional about both building deeper conceptual understanding in their students and developing a variety of algorithms that students can use with dexterity as they approach any mathematical situation or problem. When our students develop deeper understandings and greater procedural fluency, they are better able to apply those understandings and skills to other contexts, both mathematical and real-world.

## **How should teachers focus on critical areas to develop deep conceptual understanding and procedural fluency?**

"Rather than racing to cover everything in today's mile-wide, inch-deep curriculum, educators are encouraged to use the power of the eraser and significantly narrow and deepen the way time and energy is spent in the math classroom. Focus deeply on only those concepts that are emphasized in the standards so that students can gain strong foundational conceptual understanding, a high degree of procedural skill and fluency, and the ability to apply the math they know to solve problems inside and outside the math classroom."(<http://www.achievethecore.org/steal-these-tools/focus-in-math>)

Teachers should focus on the critical areas through careful planning of what to teach and when. They should consider carefully whether or not students should achieve mastery of the standard or concept, not presuming it will be taught next year. Teaching to fewer standards gives teachers more time to engage students in rigorous mathematical tasks, providing them the opportunity to both explore the concepts and construct the requisite depth of understanding. This time also allows for students to work in small groups, engage in hands-on exploration, identify patterns and analyze data, while thinking, speaking, and writing critically about their learning. Through these rigorous lessons, students will gain deep conceptual understandings and procedural fluency.

## **How is this shift represented in the CPS Mathematics Content Framework-Version 1.0?**

This shift is represented through the CPS Content Framework-Version 1.0 Planning Guides, where we lay out a carefully selected set of standards that are the focus of instruction.

In grades 6-8, this focus is based on the *Expressions and Equations* learning progression. In high school, some CCSS-M standards that address specific course content are deferred in Year 1 to provide teachers with more time to make the transition to new content. More information about this transition is provided in the CPS Bridge Plan for Mathematics (described on page 16) and in the Planning Guides.

## **Shift 2: Integrate mathematical practice standards throughout instruction**

### **What does it mean to integrate mathematical practice standards throughout instruction?**

Teachers need to explicitly incorporate the Standards for Mathematical Practice in everyday instruction. These practice standards work in concert with the content standards, as neither of them stands alone. This begins with teachers regularly using rigorous mathematical tasks (tasks with a high level of cognitive complexity), and encouraging students to grapple with these rich mathematical tasks in a way that fosters facility with the Standards for Mathematical Practice. This combination of practice and content through rich tasks and student engagement is the means by which we allow students to develop their own understanding of the mathematical content – and ultimately leads to optimal student learning gains.

### **Why is this shift so important?**

The shift to integrate the Standards for Mathematical Practice is important because the CCSS-M ask so much more from our students than prior standards. The Standards for Mathematical Practice are actually both a means to achieve the content standards and also an expectation of student performance on their own. When we are successful in implementing the Standards for Mathematical Practice, we equip students with the ability to take their mathematics learning with them into their lives and future careers. We may not know what careers are waiting for them in their future, but we do know that in our technologically advancing world we need to prepare students to think mathematically. That is, they need to be able to make mathematical sense of any given situation or problem. By using mathematical practices to master mathematical content, they will have command of a comprehensive "bag of tricks," mathematical tools and skills that will help them determine the most efficient and accurate solution, after considering a range of possibilities. In other words, mathematical practices in combination with the content standards are essential to prepare students for success.

## **How should teachers integrate mathematical practices throughout instruction?**

In mathematics classrooms that integrate the mathematical practices with content instruction, teachers implement rich

mathematical tasks and typically act as facilitators, using probing questions to stimulate students and intentionally providing students with multiple opportunities to interact directly with the content through regular mathematical discourse (either individually or in groups of students). In such a role, the teacher guides the instruction in an inquiry-based approach, in stark contrast to many direct-instruction programs in which the teacher is primarily a lecturer.

In these classrooms, the way students use practices to approach mathematical problems and arrive at an answer is just as important as arriving at the correct answer. Small-group cooperative learning may be used extensively in the mathematics classroom as an instructional strategy. The teacher, circulating among and interacting with the groups, encourages them to find ways to solve problems without giving them an example to be emulated, as in traditional classrooms. Students may also demonstrate facility by working individually (through journal writing, etc.).

With every lesson, teachers are deliberate in providing students with opportunities to develop and demonstrate mathematical practices. Through these experiences, teachers enable and encourage students to make sense of the content on their own, understanding how their previous learning informs their current tasks, and how their current tasks will build further competencies.

### **How is this shift represented in the CPS Mathematics Content Framework – Version 1.0?**

In the CPS Mathematics Content Framework’s Planning Guides, specific Standards for Mathematical Content are associated with a list of applicable Standards for Mathematical Practice. For each practice standard listed, we describe how it looks in the context of the content being taught. Additionally, the possible tasks referenced in the Planning Guides are examples of tasks that integrate and foster students’ use of the mathematical practices.

### **Shift 3: Maintain coherence and continuity to link learning within and across grade levels**

#### **What does it mean to maintain coherence and continuity of learning within and across grade levels?**

“The Common Core State Standards in mathematics were built on progressions: narrative documents describing the progression of a topic across a number of grade levels, informed both by research on children’s cognitive development and by the logical structure of mathematics.” (<http://math.arizona.edu/~ime/progressions/>) Mathematics teachers who maintain coherence and continuity of the learning progressions both (1) understand the foundation of the mathematics that led to what they are currently teaching and (2) inform their lessons with an understanding of the mathematics their students will encounter next.

#### **Why is this shift so important?**

It is extremely important for mathematics educators to carefully connect the learning within and across grades so that students can build new understandings on solid foundations. With a clear understanding of the connections between what comes before and after a particular point in the progression, teachers can address any missing prerequisite understanding or skills (revealed by assessment) and determine the next steps to move students forward from that point.

#### **How should teachers address learning progressions in their instruction?**

In 2014-2015, after the CPS transition to the CCSS-M is complete (per the CPS Bridge Plan for Mathematics, page 16), learning progressions will fully guide and inform instructional planning and delivery decisions. Teachers will be very familiar with the progressions, so they will know what mathematical content their students have learned in preceding years, and will orient instruction to that starting point. During instruction, teachers will focus on the grade-level content with an eye on future content expectations for any set of standards, or within a particular learning progression.

### **How is this shift represented in the CPS Mathematics Content Framework – Version 1.0?**

The learning progressions serve as the basis for the content standards chosen for the Planning Guides. For example, the CPS Mathematics Content Framework-Version 1.0 takes the learning progression of *Expressions and Equations* as the focus for learning in grades 6-8. This progression provides much of the foundational learning for students to prepare for the Algebra I content standards. The learning progressions and associated content standards are also reflected in the Prior Knowledge and in the sample tasks described in the Planning Guides.

## Shifts in Action: Comparing the Standards, Middle Grades

The CCSS-M guide the way for students to become capable and confident in mathematics. In raising the bar, they expose gaps in our current approaches to teaching and learning mathematics.

For example, consider how skills and practices differ in the two mathematics standards below.

| K-8 Mathematics Performance Standards  |  |
|--|--|
| Former Illinois State Learning Standard  | Common Core State Standard for Mathematics   |
| <p><b>IL STATE: (8.A.3b)</b> Solve problems using linear expressions, equations, and inequalities.</p> <p><i>Middle/Junior high school</i></p> <p><i>STATE GOAL 8: Use algebraic and analytical methods to identify and describe patterns and relationships in data, solve problems and predict results.</i></p> <p><i>A. Describe numerical relationships using variables and patterns.</i></p> | <p><b>CCSS-M: (7.EE.4)</b> Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.</p> <p><i>Grade 7</i></p> <p><i>Expressions and Equations</i></p> <p><i>Solve real-life and mathematical problems using numerical and algebraic expressions and equations</i></p> |
| <b>Standard = Content</b>  | <b>Standard = Content + Mathematical Practices</b>   |

The differences between these two standards are stark. In the former Illinois Learning Standard (ILS), the students were expected to solve a linear equation provided to them. In the CCSS-M, students are expected to define a variable, construct their own equation or inequality, and solve the problem (either in a context or not) all the while reasoning about the relationship between the quantities, the components of the equation or inequality, solving the problem and then deciding whether or not the solution is reasonable. (Mathematical Practices 1, 2 and 4)

In other words, to demonstrate mastery of the CCSS-M standard, students must apply mathematical thinking while demonstrating content knowledge. (*SHIFT 2: Integrate the mathematical practice standards throughout instruction*)

This particular standard also represents one of the clusters in the critical focus areas for grade 7 (*Solve real-life and mathematical problems using numerical and algebraic expressions and equations*). By expecting instruction to be focused in these critical areas, the standards build coherence from one grade to the next. (*SHIFT 3: Maintain coherence and continuity to link learning within and across grade levels.*)

By establishing these areas of focus, the standards eliminate some topics previously covered in the former ILS. **This is the means by which the expected depth of understanding is achieved.** (*SHIFT 1: Focus on critical areas to develop deep conceptual understanding and procedural fluency*).

## Shifts in Action: Comparing the Standards in High School

There are similar differences between standards at the high school level, as illustrated in the two mathematics standards below.

| 9-12 Mathematics Performance Standards  |   |
|---|---|
| College Readiness Standards   | Common Core State Standards for Mathematics   |
| <p><b>CRS: EEI</b> Solve real world problems using first-degree equations.<br/><i>College Readiness Standards Expressions, Equations, and Inequalities score range: 24-27</i></p> | <p><b>CCSS-M: (a-REI)</b> Solve linear equations and inequalities in one variable with coefficients represented by letters.<br/><i>Algebra Reasoning with equations and inequalities. Understanding solving equations as a process of reasoning and explaining reasoning.</i></p> |
| <b>Standard = Content</b>   | <b>Standard = Content + Mathematical Practices</b>  |

In the former College Readiness Standard (CRS) standard, students were expected to create and solve an equation for a real-world problem. In the new CCSS-M standard, the students are expected to identify the variables and quantities represented in a real-world problem and then determine the best model—linear equation, linear inequality, quadratic equation, quadratic inequality, rational equation, or exponential equation—to solve it. (*SHIFT 2: Integrate the mathematical practice standards throughout instruction*)

Students are then expected to write the equation or inequality that best models the problem and then solve the equation or inequality.

Finally, they are expected to interpret the solution in the context of the problem, per the domain. In explaining their reasoning, students demonstrate their deep understanding of the concepts and demonstrate the process for solving the mathematical task (*SHIFT 1: Focus on critical areas to develop deep conceptual understanding and procedural fluency*).

# CPS Mathematics Content Framework

The CPS Mathematics Content Framework provides the comprehensive view of how we develop mathematically capable and confident students, as defined by the CCSS-M, given the mathematics shifts in instruction described above.

Our primary objective is to provide tools and structures that will support teachers in the design of strong, school-based mathematics instruction. In the following section, we first describe District-wide expectations for the implementation of the Mathematics Content Framework-Version 1.0. Next, we describe the components of the Mathematics Toolset(s) and how they are intended to help teachers implement the CCSS-M. See below for an overview of the following sections.

Framework components include:

- District-wide Expectations
  - CPS Mathematics Bridge Plan
  - CPS Mathematics Planning Guides
  - District-wide Benchmark Performance Assessments
- Mathematics Toolsets

## District-wide Expectations

### CPS Bridge Plan for Mathematics

In 2012-2013, networks and schools will begin implementing the CPS Bridge Plan for Mathematics, the three-year blueprint that will guide the **full** implementation of the CPS Mathematics Content Framework.

The CPS Bridge Plan for Mathematics is an aggressive strategy that is intentional and deliberate in its design. It defines how we phase in new content standards and build our capacity to make the requisite shifts in instruction. In the first year of transition, we have taken into consideration that (1) there is a vast difference between the CCSS-M and the former ILS and CRS, with new content at each grade level and major shifts in instruction and approach, (2) all of the mathematics instructional materials are currently aligned to the ILS and CRS, and (3) full implementation in Year 1 would create huge gaps in student learning (students would not have the expected prior knowledge to be successful). Therefore, the Bridge Plan and other Framework components strategically focus on grades 6-8, and High School Algebra I and Geometry in Year 1 (school year 2012-13).

As you can see, by year 2014-2015 the District will be ready to implement the complete CPS Mathematics Content Framework, K-12. The following chart illustrates the three-year transition:

| Grade       | '12-'13<br>Year 1  | '13-'14<br>Year 2   | '14-'15<br>Year 3   |
|-------------|--|---|---------------------|
| K-5         | ENCOURAGED: Integrate Standards for Mathematical Practice into instruction | Implement Mathematics Content Framework 1.0 (Grades K-5)                          | 100% CCSS-M Content |
| 6-8         | Implement Mathematics Content Framework 1.0 (Grades 6-8)                   | Implement Mathematics Content Framework 2.0 (Grades 6-8)                          | 100% CCSS-M Content |
| High School | Implement Mathematics Content Framework 1.0 (Algebra I and Geometry)       | Implement Mathematics Content Framework 2.0 (Algebra I, Geometry, and Algebra II) | 100% CCSS-M Content |

## Grades K – 5

In school year 2012-13, K-5 teachers will fully implement the Common Core State Standards for Literacy (CCSS-L). In order to respect the challenges of transitioning to the CCSS-L, we are deferring the implementation of the new Common Core Standards for Mathematical Content to the following school year (2013-14). Teachers in grades K-5 are encouraged to integrate Standards for Mathematical Practice into the content that is currently being taught. In other words, teachers may adjust how students engage with the content, but not change the content itself, as a way to prepare for the 2013-14 school year.

## Grades 6 – 8

In Year 1 of the transition to CCSS-M, 2012-2013, grades 6-8 mathematics teachers will be teaching:

- to the former ILS before the ISAT in March, using their current mathematics instructional materials
- to the new CCSS-M standards, per the scope described by the Planning Guides, after the ISAT

Specifically, after the March ISAT, grades 6-8 mathematics teachers will focus on the *Expressions and Equations* learning progression (as defined in the *Progressions for the Common Core State Standards in Mathematics: 6-8, Expressions and Equations*, 2011).

This progression was intentionally chosen to build the foundation for 2012-2013 8th graders to enter the following year's high school Algebra I course ready for content that is tightly aligned with the CCSS-M. Specifically, the Grade 8 CCSS-M includes the "algebra of lines," which was formerly taught in the first half of high school Algebra I courses. ("Algebra of lines" refers to equations, graphs of linear relationships, and systems of linear equations.)

Likewise, 2012-2013 6th and 7th graders will enter the following year's mathematics classes well-prepared to succeed in classrooms where content is articulated along this critical progression. With each year in the Bridge Plan, the focus is on expanding students' cognitive development in alignment with the logical structure of the CCSS-M.

### A Look at Transition in Practice: Grades 6-8

- Through PD and "deep dives" in the *Expressions and Equations* progression, teachers will build capacity around the CCSS-M and instructional shifts
- Prior to ISAT, they apply these CCSS-M instructional shifts as they approach "old" content
  - Analyzing instructional tasks
  - Enriching them to be more aligned with CCSS-M
  - Embedding math practices
- After ISAT, teachers shift to a deeper focus on targeted CCSS-M, using guidance and resources in the Planning Guides

## Focus for Building CCSS-M Capacity, Grades 6-8

Both professional development and assessments in support of Year 1 CCSS-M implementation will be aligned with the *Expressions and Equations* content standards included in the Planning Guides.

## Algebra I and Geometry

Because the majority of CPS high schools are teaching the traditional high school mathematics course sequence, the high school mathematics Planning Guides are designed for Algebra I and Geometry courses.

- In Year 1 of the transition to CCSS-M (2012-13), the Algebra I and Geometry Planning Guides integrate both (1) the CCSS-M content standards that students did not cover in the previous school year (2011-12) **and** (2) a carefully chosen subset of the CCSS-M content expected in these courses. In this way, the Algebra I and Geometry Planning Guides address CCSS-M expectations, taking into account the expected skills and proficiencies CPS students will bring to the classroom.

- In Year 2 (2013-14), the CPS Mathematics Planning Guides will have a stronger emphasis on the CCSS-M, as students will be expected to have learned the content outlined in Year 1.

**What's in store for Algebra I, 2012-2013:** This critical course extends the mathematics that students learn in middle school. Prior to CCSS-M, the “algebra of lines” has not been fully introduced in middle school mathematics courses. (“Algebra of lines” refers to equations, graphs of linear relationships, and systems of linear equations.) The Algebra I Planning Guide includes these concepts; it does not include the Statistics and Probability conceptual category in order to provide teachers and students with the time necessary to address the content standards associated with the algebra of lines.

**What's in store for Geometry, 2012-2013:** This course formalizes and extends students' geometric experiences from the middle grades. In the Geometry Planning Guide, student learning will focus on defining geometric shapes, points, lines, and planes (concepts found in CCSS-M Grades 7 and 8). Applications of Probability will not be included in Year 1, but will be included in Year 2 (2013-2014). This focus on geometric construction is new and therefore requires more time.

### **Focus for Building CCSS-M Capacity, Algebra I and Geometry**

Both professional development and assessments in support of Year 1 CCSS-M implementation will be aligned with the following CCSS-M content standards associated with these Big Ideas in the Planning Guides (*from Appendix A: Designing High School Mathematics Courses on the Common Core State Standards* (2011)):

- Algebra I: *Quadratic Functions and Modeling; Linear and Exponential Relationships*
- Geometry: *Congruence, Proof, and Constructions; Circles With and Without Coordinates*

Please note: Although the focus for building CCSS-M capacity in high school mathematics courses centers on content standards associated with these Big Ideas as noted above, instruction should also address other content standards outlined in the Planning Guides.

### **CPS Mathematics Planning Guides**

The purpose of the CPS Mathematics Planning Guides is to provide teachers with a roadmap, to help them determine the appropriate approaches to address both the instructional shifts and the areas of instructional focus, as described in the Bridge Plan. The CPS Mathematics Planning Guides define the District’s expectations about what happens in classrooms by identifying the strategic CCSS-M standards in content and practice that CPS students will be expected to learn. Assessments, described in the next section of this document, will expect students have learned the material covered in the Planning Guides.

The Planning Guides are informed by (1) the Mathematics Model Content Frameworks developed by PARCC, the Partnership for Assessment of Readiness for College and Careers (2011), (2) the instructional shifts for mathematics, and (3) the transitional approach to implementation, as outlined in the CPS Bridge Plan for Mathematics.

## Grades 6-8 Planning Guides

### Middle School Planning Guide: Grade 8 example

## Grade 8 Mathematics Planning Guide – SY12-13

### Big Idea: Expressions and Equations

#### Big Idea Assessment: Expressions and Equations

See the 8<sup>th</sup> Grade Toolset\*: MARS task: Sorting Functions, *Summative for this Big Idea*

#### Topic: What is a Function?

##### CCSS-M Content Standards

- 8.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.
- 8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.
- 8.F.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
- 8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

| Connections to Standards for Mathematical Practice | Standards for Mathematical Practice  | How it applies...   |
|--|--|---|
|  | MP7- Look for and make use of structure.   | How different components of an equation affect or are represented in the graph of the equation; how components of the graph can lead to developing an equation  |
| Key Ideas and Terms for “What is a Function?”      | MP8 - Look for and express regularity in repeated reasoning.   | Through drawing the graphs of various equations, students will recognize the effect of different portions of the equation on the graphs; through repeating calculations, students express the generalized set of operations and create an equation.   |
|  | • Definition and characteristics of different types of functions<br>• Relationship between graph and equation<br>• Development and consistent use of the language of functions   | Function, input, output, independent variable, dependent variable, rate of change, Cartesian plane, slope, intercept, proportional<br><i>Terms should be deeply understood within the context of their use. Not to be considered standalone vocabulary exercises.</i>   |
| Prior Knowledge                                    | <ul style="list-style-type: none"> <li>• Use of letters as variables rather than boxes, etc. as in earlier grades</li> <li>• Familiarity with graphing in all four quadrants of the Cartesian plane</li> <li>• Understanding of the meaning of operations, particularly with integers</li> </ul>   |   |
| Students will be able to:                          | <ul style="list-style-type: none"> <li>• Move between tables of value and function rules to represent functions.</li> <li>• Graph functions using input-output values as ordered pairs and identify type of function through the shape of the graph.</li> <li>• Match corresponding tables, graphs, equations.</li> <li>• Draw a graph given verbal descriptions.</li> </ul> | <ul style="list-style-type: none"> <li>• Identify shapes of graphs of parent functions (linear, quadratic, cubic, exponential, etc.) and the effect of different components of the equation (leading coefficient, constant term, etc.).</li> <li>• Graph proportional relationships.</li> <li>• Compare different proportional relationships represented in different forms.</li> </ul> |
| Possible task(s)*                                  | CME Algebra 1 text, Chapter 3, for shapes of graphs of parent functions  |   |

\*The Grade 8 mathematics toolset includes this and other resources and can be found online at <https://ocs.cps.k12.il.us/sites/IKMC/default.aspx> and on the Department of Mathematics and Science website ([cmsi.cps.k12.il.us](http://cmsi.cps.k12.il.us)).

**Structure and use:** The purpose of the Planning Guides, grades 6-8, is to define the scope of the CCSS-M content standards to be taught post-ISAT, in the 2012-2013 school year. The Introduction in each Planning Guide provides essential directions for CCSS-M implementation. Within each grade-level guide, the *Expressions and Equations* learning progression, or “Big Idea,” is articulated by topics. Each topic provides instructional guidance, supported by key content standards and applicable Standards for Mathematical Practice, key terms and ideas, and sample tasks.

**Timing and instructional materials:** Schools and teachers are expected to continue using current instructional materials and to address the Illinois Assessment Frameworks from the beginning of the year until the ISAT is administered. In addition to this content, teachers will explicitly integrate the Standards for Mathematical Practice into their lessons on a regular basis.

Following the ISAT, the grades 6-8 Planning Guides form the basis for instruction in the classroom. Teachers will continue to integrate the Standards for Mathematical Practice with the content standards, in order to achieve the depth of conceptual understanding and fluency that are expected by the CCSS-M.

Please note: For many CPS middle grades classrooms, some of the CCSS-M content standards in the applicable Planning Guide will be covered within the scope of a classroom’s instructional materials prior to ISAT. We are not asking teachers to change the order of these lessons. We are asking them to teach them with the level of rigor expected by the CCSS-M, integrating the Standards for Mathematical Practice.

## High School Course Planning Guides

High School Planning Guides are specific to each traditional high school mathematics course (e.g., Algebra I, Geometry).

### High School Mathematics Planning Guide: Algebra I example

#### Algebra I Planning Guide – SY12-13

##### Big Idea: Foundations for Algebra

###### Big Idea Assessment: Foundations for Algebra

See Algebra I Toolset\*: Central Park

###### Topic: Operations on Rational Numbers

###### CCSS-M Content Standards

- 6-NS.7c Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or a negative quantity in a real-world situation.
- 7-NS.1b Understand  $p + q$  as the number located a distance of  $|q|$  from  $p$ , in the positive or negative direction depending on whether  $q$  is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
- 7-NS.1c Understand that subtraction of rational numbers as adding the additive inverse,  $p - q = p + (-q)$ . Show that the distance of two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
- 7-NS.1d Apply properties of operations as strategies to add and subtract rational numbers.
- 7-NS.2a Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as  $(-1)(-1) = 1$  and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

| Connections to Standards for Mathematical Practice | Standards of Mathematical Practice  | How it applies...   |
|--|---|---|
|  | MP2 - Reason abstractly and quantitatively.   | Manipulate the mathematical representation by showing the process considering the meaning of the quantities involved.   |
|  | MP3 - Construct viable arguments and critique the reasoning of others.  | Justify (orally and in written form) the approach used, including how it fits in the context from which the data arose.   |
|  | MP7 - Look for and make use of structure.   | Use patterns or structure to make sense of mathematics and connect prior knowledge to similar situations and extend to novel situations.  |
| Prior Knowledge                                    | <ul style="list-style-type: none"><li>The number line</li><li>Rational numbers</li><li>Additive inverse</li></ul>   | <ul style="list-style-type: none"><li>Properties of operations</li><li>Properties of operations, esp. the distributive property</li></ul>   |
| Students will be able to:                          | <ul style="list-style-type: none"><li>Find the absolute value of a rational number.</li><li>Explain the meaning of absolute value as its distance from 0 on the number line.</li><li>Add rational numbers.</li><li>Use the number line and the definition of absolute value to explain how to add rational numbers.</li><li>Explain why the sum of two opposite numbers is 0.</li><li>Apply the skill of adding rational numbers to the real world.</li></ul> | <ul style="list-style-type: none"><li>Subtract rational numbers.</li><li>Use the concepts of absolute value and additive inverse to explain how to subtract rational numbers.</li><li>Apply the skill of subtracting rational numbers to the real world.</li><li>Use properties of operations when adding and subtracting rational numbers.</li><li>Multiply rational numbers.</li><li>Interpret products of rational numbers in real world contexts.</li></ul> |
| Possible task(s)*                                  |   |   |

\*The Algebra I toolset includes this and other resources and can be found online at <https://ocs.cps.k12.il.us/sites/IKMC/default.aspx> and on the Department of Mathematics and Science website ([cmsi.cps.k12.il.us](http://cmsi.cps.k12.il.us)).

**Structure and use:** Each high school mathematics course Planning Guide describes the scope of CCSS-M content standards to be taught in the 2012-2013 school year. The Introduction in each Planning Guide provides essential directions for CCSS-M implementation. Each course is broken down into Big Ideas. A performance task, or problem, is referenced that illustrates what students will know and be able to do by the successful completion of the Big Idea. Within each Big Idea, topics provide more instructional guidance, supported by key content standards and associated mathematical practices.

**Timing and instructional materials:** Teachers are expected to cover the entire scope of content outlined in the high school mathematics course Planning Guides, in the order most appropriate to their students, instructional materials, and learning environments.

Using their current instructional materials, teachers will integrate the Standards for Mathematical Practice with the content standards, and enhance their instruction with rich mathematical tasks in order to achieve the depth of conceptual understanding and fluency that are expected by the CCSS-M. Tools to analyze tasks for rigor, sample rigorous mathematical tasks, and other resources to support lesson enhancement are included in the Planning Guide toolsets.

## District-wide Benchmark Performance Assessments

Assessments are critical to a successful transition to the CCSS-M. In order to prepare our students for the rigor defined by the CCSS-M, our assessments must be designed to measure the conceptual understanding and the mathematical thinking expected of the CCSS-M. This is also the design of the upcoming Partnership for Assessment of Readiness of College and Career (PARCC) Assessment system for mathematics, which will be the accountability measure for CPS in 2014-15. PARCC tests will be designed to measure the knowledge, skills and understandings essential to achieving college and career readiness – as defined by the CCSS-M. To measure the full range of the standards, all assessments will include tasks that require students to connect mathematical content and mathematical practices. (*PARCC Model Content Frameworks for Mathematics, October 2011*)

The CPS Mathematics Performance Assessments will help students prepare for the PARCC tests by assessing material described in the Planning Guides. More importantly, these kinds of assessments mimic tasks that students will be expected to perform in college and/or career.

### Performance Assessment for Grades K-5

BOY: These assessments will not be provided centrally by CPS.

During Year: OPTIONAL assessments based on MARS Tasks\*, TBD

EOY: These assessments will not be provided centrally by CPS.

### Performance Assessment for Grades 6-8

BOY: Pre-assessment of CCSS-M standards aligned with the *Expressions and Equations* learning progression\*\*

During Year: Formative Assessments options based on MARS Tasks\*, TBD

EOY: Post-assessment of CCSS-M standards aligned with the *Expressions and Equations* learning progression\*\*

### Performance Assessment Provided by District for High School Algebra I and Geometry

Will be aligned to the focus for building CCSS-M capacity for Algebra I and Geometry, Year 1:

- Algebra I: *Quadratic Functions and Modeling; Linear and Exponential Relationships*
- Geometry: *Congruence, Proof, and Constructions; Circles With and Without Coordinates*

BOY: Pre-assessment of CCSS-M standards aligned with the focus areas of learning for Algebra I and Geometry, respectively\*\*

During Year: Formative Assessments options based on MARS Tasks\*, TBD

EOY: Post-assessment of CCSS-M aligned with the focus areas of learning for Algebra I and Geometry, respectively\*\*

\* The MARS (Mathematics Assessment Resource Service) tasks demand substantial chains of reasoning and non-routine problem solving. Most MARS tasks are designed to be accessible to most learners: the tasks begin with questions slightly below the level of difficulty of the grade level, and are meant to lead students into the concept and activate their prior knowledge. The tasks increase in rigor and difficulty such that teachers are able to determine student misconceptions.

\*\* These performance tasks will also be used to measure student growth for teacher evaluation.

# CPS Mathematics Toolsets

To support our transition to CCSS-M, we have developed tools for teachers to use as they plan and implement instruction aligned with CCSS-M expectations. These tools are described below and can be found at <https://ocs.cps.k12.il.us/sites/IKMC/default.aspx> and also at <http://cmsi.cps.k12.il.us>.

- Sample Lesson Plan Template
- Samples of Rigorous Tasks (for each Planning Guide, Grades 6-8, Algebra I, and Geometry)
- Tool for Examining and Modifying Lessons/Tasks
- Samples of Modified Lessons/Tasks
- Recommended Professional Resources

## Sample Lesson Plan Template

The Lesson Plan Template provides a structure to guide teachers as they think through the components of a CCSS-M aligned lesson. These components include: the topic; standards (practice and content) and evidence of them; lesson flow and timing; potential facilitation questions; and anticipated potential misconceptions and how to address them.

## Sample Rigorous Tasks at each grade level

Within each Planning Guide, rigorous performance tasks, or problems, are referenced at both the Big Idea and topic levels. These can be found at <https://ocs.cps.k12.il.us/sites/IKMC/default.aspx> and also at <http://cmsi.cps.k12.il.us>.

## Lesson Analysis and Modification Tool

This tool is a rubric with an associated form with criteria to help teachers look at a lesson from their current materials and decide how well it is aligned to CCSS-M expectations (e.g., depth of conceptual understanding, integration of Standards for Mathematical Practice, focused on content standards, etc.). If a lesson is not well-aligned, teachers use the tool's criteria to increase the rigor of the lesson.

## Samples of Modified Lessons/Activities

These samples include (1) an original lesson, (2) the completed analysis tool (above), and (3) the final lesson as it was modified to address the shortcomings identified through the analysis.

## Recommended Professional Resources

A list of vetted resources that will be helpful to teachers as they transition to CCSS-M in the classroom, which include: sample tasks and lessons, standards clarification and guidance, and pedagogical support. Many are free; others we recommend because we feel they are worth the price. The list will be updated as more tools and resources are developed, as more cities and states transition to CCSS-M.

# Getting Started

When teachers meet with their grade-level and/or course teams to develop a CCSS-M plan, how should they begin? The goal is for teacher teams to work together to determine how to implement the CCSS-M and instructional shifts, as described by this document – while considering the context of their school (teachers, students, instructional materials, etc.). The following planning outline will help all teams begin the transition to CCSS-M in their classrooms.

1. Become familiar with the CPS Mathematics Content Framework-Version 1.0 (this document), including the instructional shifts and the CPS Bridge Plan for Mathematics.
2. Use the CCSS-M materials to become familiar with Standards for Mathematical Practice. Teams are encouraged to work together to develop new instructional approaches that support these practice standards.
3. Become familiar with:
  - a. The kinds of high-cognitive demand tasks that are expected by the CCSS-M
  - b. Ways to analyze tasks in current instructional materials for rigor
  - c. Techniques to enhance the rigor of current instructional materials. In the appendix of this document, there are general resources and tools that support the kind of learning expected in the CCSS-M. Tools in the Planning Guides include grade- or course-specific resources, including sample MARS tasks. These tasks balance content and practice, for an integrated approach to instruction and performance assessment. In addition, general mathematics tools to support rigorous instruction are available online at <https://ocs.cps.k12.il.us/sites/IKMC/default.aspx> and on the Department of Mathematics and Science website ([cmsi.cps.k12.il.us](http://cmsi.cps.k12.il.us)).
4. If you teach grades 6-8 mathematics, follow **Getting Started - A**
5. If you teach Algebra 1 or Geometry, follow **Getting Started - B**
6. If you teach mathematics in grades K-5, or any high school course other than Algebra 1 and Geometry—in other words, outside the scope of Year 1—follow **Getting Started - C**.

## Things to Remember

- The planning process is iterative. In other words, as you plan for instruction, an intentional effort should be always be made to incorporate the instructional shifts in all lessons. Though we are gradually phasing in the Standards for Mathematical **Content** (in grades 6-8, Algebra I, and Geometry), the Standards for Mathematical **Practice** should be integrated in all mathematics instruction, per the Year 1 CPS Bridge Plan for Mathematics. Use the Planning Guides and Mathematics Toolsets to inform your planning and instruction.
- The most important parts of the process will be your instruction, collaboration with colleagues, and ongoing reflection about how your students are doing. As you implement your plans, continue to meet with your colleagues to study students' work and revise instruction accordingly.
- Never underestimate your professional judgment. Your knowledge of your students and their needs should always be the forerunner in your planning.

## GETTING STARTED A: For grades 6-8 (Year 1 of Bridge Plan)

### Guidance

1. Consider the three instructional shifts that teachers must implement in order to support student success in meeting the CCSS-M. What instructional strategies are already being used that support these shifts? What adjustments to instruction are needed to address the instructional shifts? Integrating the Standards for Mathematical Practice into instruction may involve substantial shifts in instructional strategies.
2. Use the CCSS-M materials to become familiar with the content standards associated with the *Expressions and Equations* learning progression.
3. Become familiar with the Planning Guide and toolset for your grade level.
4. Before ISAT, plan to enhance your current instructional materials at the points where the *Expressions and Equations* content standards (from Planning Guide) are addressed, to provide the rigor expected by the CCSS-M. Use the resources referenced in appendix (in this document) and in the Mathematics toolset to analyze and modify your lessons and tasks.
5. After ISAT, use the Planning Guide to inform your planning and instruction. If your materials did not cover the expected *Expressions and Equations* content standards before ISAT, use the Planning Guide to plan instruction for remaining time.
6. Construct (or revise) your plans by addressing each component laid out in the Planning Guide.
  - a. What other standards need to be revisited or introduced to (1) support any missing skills/ knowledge? And (2) develop deep conceptual understandings and procedural fluency?
  - b. What tasks will help build the skills and knowledge your students need in order to master the key content standards, or be prepared for the next content standards in learning progression?
  - c. How can you supplement activities in your instructional materials to be more rigorous? High quality tasks will ignite student learning and provide a solid foundation upon which to build more complex mathematics.
  - d. How will you integrate the Standards for Mathematical Practice? Adjust your instructional strategies to enable and encourage students to make sense of the content on their own?
  - e. How can you use formative assessments and scoring tools to align with your Planning Guide and specific topics and tasks?

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## GETTING STARTED B: For Algebra 1 and Geometry (Year 1 of Bridge Plan)

### Guidance

1. Consider the three instructional shifts that teachers must implement in order to support student success in meeting the CCSS-M. What instructional strategies are already being used that support these shifts? What adjustments to instruction are needed to address the instructional shifts? Integrating the Standards for Mathematical Practice into instruction may involve substantial shifts in instructional strategies.
2. Use the CCSS-M materials (including Appendix A) to become familiar with the content standards associated with the focus areas for building CCSS-M capacity in Year 1.
  - a. Algebra I: *Quadratic Functions and Modeling; Linear and Exponential Relationships*
  - b. Geometry: *Congruence, Proof, and Constructions; Circles With and Without Coordinates*
3. Become familiar with the detailed expectations for learning in your course Planning Guide and the toolset to support your instruction. The scope described in the Planning Guide will lay the foundation for student success in Year 2 of our transition to CCSS-M.
4. Construct (or revise) your plans by addressing each component laid out in the Planning Guide.
  - a. Considering your students, instructional materials, and other factors, in what order will you present the Big Ideas? Against what timetable?
  - b. What other content standards need to be revisited or introduced to (1) support any missing skills/ knowledge? And (2) develop deep conceptual understandings and procedural fluency?
  - c. What tasks will help build the skills and knowledge your students need in order to master the key content standards, or be prepared for the next content standards outlined in the next Big Idea you will cover?
  - d. How can you supplement activities in your instructional materials to be more rigorous? High quality tasks will ignite student learning and provide a solid foundation upon which to build more complex mathematics.
  - e. How will you integrate the Standards for Mathematical Practice? Adjust your instructional strategies to enable and encourage students to make sense of the content on their own?
  - f. How can you use formative assessments and scoring tools to align with your Planning Guide and specific topics and tasks?

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## GETTING STARTED C: For grades K-5, 11-12 mathematics classrooms (outside scope of Year 1 of Bridge Plan)

### Guidance

1. Use your current instructional materials as a starting point.
2. To the extent possible, integrate the Standards for Mathematical Practice into instruction. Every teacher is strongly encouraged to apply the Standards for Mathematical Practice into every CPS mathematics classroom and activity even though teachers may not be implementing the content standards this year. Integrating mathematical practices into instruction may involve substantial shifts in instructional strategies (such as the art of open-ended questioning, as referenced in the Shifts, page 11)
3. Begin to increase rigor of mathematical tasks. Use the resources in the Appendix to assess your lessons for rigor and supplement your current instructional materials, as appropriate.
4. Consider how to upgrade assessments to reflect the work you are doing with the Standards for Mathematical Practice. For example, choose problems with higher cognitive demand, such as open-ended or extended/constructed response.

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# Implementation of CCSS-M

## Planning with ALL Learners in Mind

The Common Core State Standards were written for ALL K-12 students and, as such, effective implementation of the Standards rests on the intentional planning of instruction to provide access to learning for all students.

Student diversity is always present whether recognized or not; it is a given in every classroom. Within every group of students, teachers can anticipate that there will be a variety of skills, affinities, challenges, experiences, cultural lenses, aptitudes, interests, English language proficiency levels, (in the case of ELLs, native language proficiency levels), represented. As teachers engage in initial stages of curriculum planning (i.e. the clustering of standards, selection of texts and tasks, and the design of performance assessments) they must simultaneously consider the variety of learner profiles among their students. It is critical that teachers start curriculum planning with both the Standards and the Learners in mind.

Intentional planning for a diverse student group from the outset will maximize the likelihood that all students will be able to successfully access information, process concepts, and demonstrate their learning. Early in the year or course, data from various sources such as cumulative folders, screeners, pre-tests, Individualized Education Plans (IEPs), Individual Bilingual Instruction Plans (IBIPs), parent questionnaires, and getting-to-know-you activities, give teachers important preliminary information about every individual student that will influence their plans. As teachers better get to know individual students and their particular learning needs, over time they can continuously adjust curricular plans and personalize instructional strategies for more tailored differentiation.

Having initial plans that are universally-designed will position teachers to serve most students well, but in the process of personalizing the plan and as teachers would know, there will be certain elements that are crucial to include explicitly for particular groups of students. For example, while every child is unique and will therefore benefit from attention to their individual learner profile, a student who has been identified with a disability, by law and best practice, will require instructional supports based upon the IEP team's best thinking relative to academic and functional need. Similarly, while every child is in the process of developing language and will therefore benefit from an educational experience that is designed for a range of social and academic English levels, a student who has been identified as an English Language Learner (ELL), by law and best practice, will have needs that must be addressed in particular ways. In both cases, it is important for teachers to specifically recognize and articulate in their curriculum plans and in their methods of instruction how they will tailor learning for these individuals.

## Resources for Addressing Universal Variability

|   |  |
|---|--|
| Article 14, Illinois School Code, Title 23, Administrative Code.226           | Establishes the requirements for the treatment of children and the provision of special education and related services pursuant to the Individuals with Disabilities Education Improvement Act.<br><br><a href="http://www.isbe.net/rules/archive/pdfs/226ark.pdf">http://www.isbe.net/rules/archive/pdfs/226ark.pdf</a> |
| Article 14C, Illinois School Code, Title 23 Illinois Administrative Code. 228 | Establishes requirements for school districts' provision of services to students in preschool through grade 12 who have been identified as limited English proficient.<br><br><a href="http://www.isbe.net/rules/archive/pdfs/228ark.pdf">http://www.isbe.net/rules/archive/pdfs/228ark.pdf</a>                          |
| CAST  | CAST is a nonprofit research and development organization that works to expand learning opportunities for all individuals, especially those with disabilities, through Universal Design for Learning.<br><br><a href="http://www.cast.org">www.cast.org</a>  |
| Illinois Resource Center  | The IRC provides assistance to teachers serving linguistically and culturally diverse students, including English language learners (ELLs), in grades PK-12.<br><br><a href="http://www.thecenterweb.org/irc">www.thecenterweb.org/irc</a>   |
| National Center on Universal Design for Learning                              | The National UDL Center supports the effective implementation of UDL by connecting stakeholders in the field and providing resources and information about: UDL Basics, Advocacy, Implementation Research, Community, Resources.<br><br><a href="http://www.udlcenter.org">www.udlcenter.org</a>                         |
| STAR NET Region II  | STAR NET Region II provides technical assistance, training and resources to professionals and families supporting the education of young children, with an emphasis on children with special needs.<br><br><a href="http://www.thecenterweb.org/starnet">www.thecenterweb.org/starnet</a>                                |
| Understanding Language  | Understanding Language aims to heighten educator awareness of the critical role that language plays in the new Common Core State Standards and Next Generation Science Standards.<br><br><a href="http://www.ell.stanford.edu">www.ell.stanford.edu</a>  |
| World-Class Instructional Design and Assessment                               | WIDA advances academic language development and academic achievement for linguistically diverse students through high quality standards, assessments, research, and professional development for educators.<br><br><a href="http://www wida us">www wida us</a>  |

## Works Cited

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## Planning Guides

The following Planning Guides outline the scope of learning for grades 6-8 and Algebra I and Geometry for the 2012-2013 school year. These Planning Guides provide guidance to help teachers build deep conceptual understanding of mathematics, procedural fluency, and critical thinking and problem-solving skills. <https://ocs.cps.k12.il.us/sites/IKMC> and on the Department of Mathematics and Science website: <http://cmsi.cps.k12.il.us/>.



# Grade 6 Mathematics Planning Guide – SY12-13

## Introduction

The purpose of this Planning Guide is to define the scope of the Common Core State Standards for Mathematics (CCSS-M) Content Standards to be taught in the 2012-2013 school year. It has been designed by CPS teachers to be useful to CPS teachers during the three-year transition to full implementation of the CCSS-M.

In Year 1 of the transition to CCSS-M, 2012-2013, grades 6-8 mathematics teachers will be teaching:

- **To the former ILS before the ISAT in March.** The expectation is for schools and teachers to continue using their current mathematics instructional materials and to address the Illinois Assessment Frameworks from the beginning of the year until the ISAT is administered. In addition to this content, teachers will explicitly integrate the Standards for Mathematical Practice into their lessons on a regular basis.
- **To the new CCSS-M standards, per the scope described by this Planning Guide, after the ISAT,** using their instructional materials and other resources to integrate mathematical practices with rigorous mathematical tasks that support the scope of content standards. A toolset to support this instruction includes (1) samples of rigorous grade-specific tasks (including samples of formative assessment options based on MARS (Mathematics Assessment Resource Service) tasks) and (2) general tools that include a sample lesson planning template, a tool for analyzing and modifying lessons/tasks; samples of modified lessons/tasks; and a list of professional resources. These tools are available at <https://ocs.cps.k12.il.us/sites/IKMC/default.aspx> and on the Department of Mathematics and Science website ([cmsi.cps.k12.il.us](http://cmsi.cps.k12.il.us)).

Specifically, after the March ISAT, grades 6-8 mathematics teachers will focus on the *Expressions and Equations* progression, as defined in the *Progressions for the Common Core State Standards in Mathematics: 6-8, Expressions and Equations* (2011). This progression was chosen because the Grade 8 CCSS-M includes the “algebra of lines,” which was formerly taught in the first half of high school Algebra I courses (“algebra of lines” refers to equations, graphs of linear relationships, and systems of linear equations). Concentrating on this particular progression allows grades 6-8 classrooms to hone in on the vertical articulation of this very important concept, focusing on students’ cognitive development and the logical structure of the CCSS-M.

Please note: If the content standards in this Planning Guide are covered within the scope of a classroom’s instructional materials prior to ISAT, teachers should teach to these with the level of rigor expected by the CCSS-M, integrating the Standards for Mathematical Practice with the content. Post-ISAT is an opportunity to reinforce content standards already addressed, and to focus more deeply on the *Expressions and Equations* content standards that were not covered prior to ISAT.

During the first 3 quarters, teachers will explicitly integrate the Standards for Mathematical Practice (below) into their lessons on a regular basis. In the 4th quarter, instruction that integrates Standards for Mathematical Practice with targeted content standards is supported by “how to” guidance in this Planning Guide. By teaching the mathematical practices alongside the indicated content standards, students will be more likely to achieve the depth of conceptual understanding and procedural fluency expected by the CCSS-M.

## The CCSS-M Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Following the ISAT, this Planning Guide will form the basis for instruction in Grade 6 mathematics classrooms. This year's focus of learning, *Expressions and Equations*, is broken into four topics: 1) Using Ratio Reasoning to Solve Problems, 2) Algebraic Expressions, 3) Single Variable Expressions, Equations, and Inequalities, and 4) Graphing in the Coordinate Plane. Topic components include a targeted set of content standards, target mathematical practices (and how they apply to the specific topic), key ideas for learning, key terms, and sample instructional tasks.

The guide assumes 9 weeks for instruction, including time for formative and summative assessments. The topics are sequenced in a way that we believe best develops and connects the mathematical content of the CCSS-M. However, teachers should review the topics and decide the order and time allocation appropriate for their classrooms, given their students, instructional materials, and other considerations. The order of the standards included in a topic does not imply a sequence of the content. Some standards may be revisited several times while addressing the topic, while others may be only partially addressed, depending on the mathematical focus of the topic.

Finally, this document reflects our current thinking about the transition to the CCSS-M. We welcome feedback about your experience with the document. Please share your thoughts with your network staff who will forward to the Department of Mathematics and Science.

# Grade 6 Mathematics Planning Guide – SY12-13

## Big Idea: Expressions and Equations

### Big Idea Assessment: Expressions and Equations

See 6<sup>th</sup> Grade Toolset\*: Sample 6<sup>th</sup> Grade assessment (collection of problems from Illustrative Math)

#### Topic: Using Ratio Reasoning to Solve Problems

##### CCSS-M Content Standards

- 6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.  
6.RP.2 Understand the concept of a unit rate  $a/b$  associated with a ratio  $a:b$  with  $b \neq 0$ , and use rate language in the context of a ratio relationship.  
6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams or equations.
- Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
  - Solve unit rate problems including those involving unit pricing and constant speed.
  - Find a percent of a quantity as a rate per 100; solve problems involving finding the whole, given a part and the percent.
  - Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

##### Connections to Standards for Mathematical Practice for Mathematical Practice

MP2 – Reason abstractly and quantitatively.

MP4 – Modeling with mathematics.

##### Standards for Mathematical Practice

- Especially in unit conversion (6.RP.3d) - creating a symbolic representation of the problem and contextualizing the result of the calculations back to the context of the problem.
- Students are able to identify important quantities in a practical situation and map their relationships using diagrams, two way tables, graphs, flow-charts, etc. and use this info to derive a formula.

##### Key Ideas and Terms for “Using Ratio Reasoning to Solve Problems”

- Understand and use rational reasoning to calculate a unit rate
- Apply a ratio to calculate the composition of a set
- Use ratio reasoning to derive a generalized description of a set

##### Key Ideas

- Ratio, proportion, unit rate, equivalent terms should be deeply understood within the context of their use. Not to be considered standalone vocabulary exercises.
- Use equivalent fractions as a strategy to add and subtract fractions
  - Multiply and divide fractions
  - Simplify fractions into lowest terms

##### Prior Knowledge

- Create and describe a ratio relationship between two quantities.
- Express ratios in different forms.
- Understand and derive unit rate.
- Determine equivalent ratios.
- Create tables of equivalent ratios.
- Use ratio relationships to determine percent, part, or whole given the others.
- Apply ratio reasoning when converting units of measurement.

##### Possible tasks\*

MARS Task: Candies; Elevators; Boys and Girls Task; Sugar Cookies; Shades of Blue

\*The Grade 6 mathematics toolset includes this and other resources and can be found online at <https://ocs.cps.k12.il.us/sites/IKMC/default.aspx> and on the Department of Mathematics and Science website ([cmstools.cps.k12.il.us](http://cmstools.cps.k12.il.us)).

# Grade 6 Mathematics Planning Guide – SY12-13

## Big Idea: Expressions and Equations

### Topic: Algebraic Expressions

#### CCSS-M Content Standards

- 6.EE.1 Write and evaluate numerical expressions involving whole-number exponents
- 6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers
- 6.EE.3 Apply the properties of operations to generate equivalent expressions.
- 6.EE.4 Identify when two expressions are equivalent.

| Connections to Standards for Mathematical Practice | Standards for Mathematical Practice  | How it applies...  |
|--|--|--|
|  | MP2 - Reason abstractly and quantitatively.  | When evaluating expressions ( $3x + 7$ when $x=4$ ) students are <i>decontextualizing</i> the information but then must interpret the result as it applies to the context of the problem.  |
|  | MP7 - Look for and make use of structure.  | When comparing two expressions, students must understand the structure of the two (variables, coefficients, exponents) in order to determine equivalence.  |
|  | MP8 – Look for and express regularity in repeated reasoning.   | This is a powerful practice when moving from numerical and arithmetic reasoning to algebraic representation. By attending to calculations when solving specific instances of a problem, students will be able to generalize the process and write an algebraic expression.     |
| Key Ideas and Terms for “Algebraic Expressions”    | Key Ideas  | Key Terms  |
|  | <ul style="list-style-type: none"><li>• Apply arithmetic operations to variable expressions</li><li>• Build algebraic expressions from a context</li></ul>   | <p>Expression, evaluate, variable, equivalent, coefficient, quotient, sum, term, product, factor, distributive property, simplified</p> <p><i>Terms should be deeply understood within the context of their use. Not to be considered standalone vocabulary exercises.</i></p> |
| Prior Knowledge                                    |  |  |
|  | <ul style="list-style-type: none"><li>• Fluency with arithmetic operations and their connections to real world contexts</li><li>• Experience in simplifying numerical expressions following the order of operations</li><li>• Translate from a verbal description of a list of operations to an arithmetic representation with numbers</li><li>• Fluency with mathematical terms representing operations</li></ul> |  |
| Students will be able to:                          |  |  |
|  | <ul style="list-style-type: none"><li>• Create and evaluate numerical expressions.</li><li>• Create algebraic expressions from word expressions.</li><li>• Create algebraic expressions from context.</li><li>• Use mathematical terms to describe expressions.</li></ul>  | <ul style="list-style-type: none"><li>• Apply the distributive property and other operations to algebraic expressions to determine equivalence.</li><li>• Simplify algebraic expressions.</li><li>• Recognize equivalent expressions.</li></ul>                                |
| Possible tasks*                                    | Illuminations (NCTM) Building Bridges; Valentine’s Cards   |  |

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# Grade 6 Mathematics Planning Guide – SY12-13

## Big Idea: Expressions and Equations

### Topic: Single Variable Expressions, Equations, and Inequalities

CCSS-M Content Standards

- 6.EE.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
- 6.EE.6 Use variables to represent numbers and write expressions when solving real-world or mathematical problems; understand that a variable can represent an unknown , or, depending on the purpose at hand, any number in a specified set.
- 6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form  $x + p = q$  and  $px = q$  for cases in which  $p$ ,  $q$ , and  $x$  are all nonnegative rational numbers.
- 6.EE.8 Write an inequality of the form  $x > c$  or  $x < c$  to represent a constraint or condition in a real world or mathematical problem. Recognize that inequalities of the form  $x > c$  have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

| Connections to Standards for Mathematical Practice                                 | Standards for Mathematical Practice   | How it applies...  |
|--|---|--|
|  | MP4 – Modeling with mathematics.  | Creating a generalized representation of a situation (an equation or inequality)   |
|  | MP7 – Look for and making use of structure.   | Understanding and using a “variable” as a placeholder for an unknown in a series of calculations and then undoing those calculations to find the value of the unknown. Analyzing a variable expression on one side of an equation and recognizing that it represents the number.   |
| Key Ideas and Terms for “Single Variable Expressions, Equations, and Inequalities” | Key Ideas   | Key Terms  |
|  | <ul style="list-style-type: none"><li>Reason about simple single variable expressions</li><li>Reason about (and solve) simple single variable equations</li><li>Reason about (and solve) simple single variable inequalities</li><li>Understand that a solution is a value that when substituted for a variable, makes the equation or inequality true.</li></ul> | <p>Equation, inequality, set, substitution, variable, solution<br/><i>Terms should be deeply understood within the context of their use. Not to be considered stand-alone vocabulary exercises.</i></p> <ul style="list-style-type: none"><li>Translate from a verbal description of a list of operations to an arithmetic representation with numbers</li><li>Fluency with mathematical terms representing operations</li></ul> |
| Prior Knowledge  | <ul style="list-style-type: none"><li>Fluency with arithmetic operations and their connections to real world contexts</li><li>Experience in simplifying numerical expressions following the order of operations</li></ul>   |  |
| Students will be able to:  | <ul style="list-style-type: none"><li>Create algebraic expressions from context.</li><li>Create and solve algebraic equations from context.</li><li>Create inequalities to represent a constraint or condition from context.</li></ul>  | <ul style="list-style-type: none"><li>Use substitution to determine if a solution makes an inequality true</li><li>Represent solutions of inequalities on number line diagrams</li></ul>   |
| Possible tasks*  | From Cognitive Tutor Algebra 1: Computer Games, CDs, and DVDS; Selling Cars; A Park Ranger’s Work is Never Done   |  |

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# Grade 6 Mathematics Planning Guide – SY12-13

## Big Idea: Expressions and Equations

### Topic: Graphing in the Coordinate Plane

#### CCSS-M Content Standards

- 6.NS.8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

| Connections to Standards for Mathematical Practice         | Standards for Mathematical Practice  | How it applies...   |
|--|--|---|
|  | MP4 – Model with mathematics.  | Using a graph to represent a set of data creates a visual model that allows for drawing generalized conclusions   |
| Key Ideas and Terms for “Graphing in the Coordinate Plane” | Key Ideas  | Key Terms   |
|  | <ul style="list-style-type: none"><li>Graphing related pairs of values in order to understand or represent the relationship between the quantities</li></ul>   | Coordinate plane, quadrant, x-axis, y-axis, origin, absolute value<br><i>Terms should be deeply understood within the context of their use. Not to be considered standalone vocabulary exercises.</i> |
| Prior Knowledge  | <ul style="list-style-type: none"><li>Graph points on the coordinate plane in the first quadrant (5.G.1 &amp; 5.G.2)</li></ul>   |   |
| Students will be able to:                                  | <ul style="list-style-type: none"><li>Graph ordered pairs in any quadrant.</li><li>Determine the distance between two coordinates aligned vertically or horizontally.</li><li>Use absolute value to find the distance between two coordinates and recognize absolute value as a way to account for direction when determining distances.</li></ul> |   |
| Possible tasks*  | Coordinate Plane Pictures; CME Algebra 1 lessons 3.1 and 3.3   |   |

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4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Following the ISAT, this Planning Guide will form the basis for instruction in Grade 7 mathematics classrooms. This year's focus of learning, Expressions and Equations, is broken into three topics: 1) Linear Relationships, 2) Modeling Linear Relationships, and 3) Analyzing Linear Relationships. Topic components include a targeted set of content standards, target mathematical practices (and how they apply to the specific topic), key ideas for learning, key terms, and sample instructional tasks.

The guide assumes 9 weeks for instruction, including time for formative and summative assessments. The topics are sequenced in a way that we believe best develops and connects the mathematical content of the CCSS-M. However, teachers should review the topics and decide the order and time allocation appropriate for their classrooms, given their students, instructional materials, and other considerations. The order of the standards included in a topic does not imply a sequence of the content. Some standards may be revisited several times while addressing the topic, while others may be only partially addressed, depending on the mathematical focus of the topic.

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# Grade 7 Mathematics Planning Guide – SY12-13

## Big Idea: Expressions and Equations

### Big Idea Assessment: Expressions and Equations See 7<sup>th</sup> Grade Toolset\*: Fishing Adventures and Sales Tax

#### Topic: Linear Relationships

##### CCSS-M Content Standards

- 7.EE.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients  
7.EE.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.  
7.NS.1d Apply properties of operations as strategies to add and subtract real numbers.  
7.NS.1c Understand subtraction of rational numbers as adding the additive inverse,  $p - q = p + (-q)$ . Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.  
7.NS.2a Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as  $(-1)(-1) = 1$  and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

| Connections to Standards for Mathematical Practice | Standards for Mathematical Practice   | How it applies...   |
|--|---|---|
|  | MP2 - Reason abstractly and quantitatively.   | Connecting different components of an expression to the context it represents and transforming an expression so that it reveals the desired information.  |
|  | MP4 - Model with mathematics.   | Creating an algebraic expression or equation that represents a situation or problem presented   |
|  | MP7 - Look for and make use of structure.   | Viewing $3(x+12)$ as 3 of something called $(x + 12)$ provides understanding of how to expand the expression to $3x + 36$ which allows for interpretation of the 3 as a unit rate or slope.   |
| Key Ideas and Terms for “Linear Relationships”     | Key Ideas   | Key Terms   |
|  | <ul style="list-style-type: none"><li>Apply knowledge of arithmetic operations and properties of numbers to expressions with variables to create different but equivalent expressions</li><li>Create and interpret equivalent expressions</li></ul>   | <p>Expression, equivalent, expand, simplify, rational number, distributive property</p> <p>Terms should be deeply understood within the context of their use. Not to be considered standalone vocabulary exercises.</p>   |
| Prior Knowledge                                    | <ul style="list-style-type: none"><li>Fluency with arithmetic operations and their connections to real world contexts</li><li>Experience in simplifying numeric expressions following the order of operations</li><li>Solving proportions using ratio and rate reasoning to extend a pattern</li><li>Familiarity with the number line</li></ul> |   |
| Students will be able to:                          | <ul style="list-style-type: none"><li>Expand linear expressions with rational coefficients.</li><li>Create and interpret equivalent expressions, linking them to the context of the problem.</li><li>Understand absolute value as distance between values and apply it to real world contexts.</li></ul>  | <ul style="list-style-type: none"><li>Add, subtract, multiply, and divide rational numbers.</li><li>Understand and apply the distributive property to expand rational expressions.</li><li>Model numerical expressions on the number line.</li></ul>  |
| Possible Tasks*                                    | MARS Task: “Pedro’s Tables”; MAP Task A12: “Fencing”; Illustrative Mathematics Task: 7EE: “Discounted Books” and “Equivalent Expressions”   | * The Grade 7 mathematics toolset includes this and other resources and can be found online at <a href="https://ocs.cps.k12.il.us/sites/IKMC/default.aspx">https://ocs.cps.k12.il.us/sites/IKMC/default.aspx</a> and on the Department of Mathematics and Science website ( <a href="http://cmi.cps.k12.il.us">cmi.cps.k12.il.us</a> ). |

# Grade 7 Mathematics Planning Guide – SY12-13

## Big Idea: Expressions and Equations

### Topic: Modeling Linear Relationships

#### CCSS-M Content Standards

- 7.EE.4a Solve word problems leading to equations of the form  $px + q = r$  and  $p(x + q) = r$  where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.
- 7.EE.4b Solve word problems leading to inequalities of the form  $px + q > r$  or  $px + q < r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Graph the solution set of inequalities and interpret them in the contexts of the problems.
- 7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.
- 7.RP.2 Recognize and represent proportional relationships between quantities.

| Connections to Standards for Mathematical Practice      | Standards for Mathematical Practice  | How it applies...  |
|---|--|--|
|   | MP1 – Make sense of the problem and persevere in solving it.   | Through defining what the word problems are asking (7.EE.4a/b); comparing an algebraic solution to an arithmetic solution (7.EE.3) and assessing the reasonableness of an answer (7.EE.3)  |
|   | MP2 – Reason abstractly and quantitatively.  | <i>Decontextualizing</i> - creating an equation or expression that gets transformed and then <i>contextualizing</i> - connecting the new expression or equation back into the context of the problem   |
|   | MP4 – Model with mathematics.  | Creating an equation from the context of a word problem.   |
| Key Ideas and Terms for “Modeling Linear Relationships” | Key Ideas  | <b>Key Terms</b><br>Unit rate, solution, solution set, multi-step problems, estimation, approximation, ratio, proportion, scale factor, complex fractions<br><i>Terms should be deeply understood within the context of their use. Not to be considered standalone vocabulary exercises.</i>   |
|   | <ul style="list-style-type: none"> <li>Model/Represent contextual linear problems with:<br/>single variable expressions, equations, inequalities, tables, and graphs</li> </ul>  |  |
| Prior Knowledge   | <ul style="list-style-type: none"> <li>Knowledge of and capability to plot points in all four quadrants of the coordinate plane</li> <li>Understand rates in the forms of equivalent ratios and tables</li> <li>Write, read, and evaluate expressions in which letters stand for numbers</li> </ul>                                | <ul style="list-style-type: none"> <li>Identify when two expressions are equivalent</li> <li>Understand that positive and negative numbers are used together to describe quantities having opposite directions or values</li> </ul>  |
| Students will be able to:                               | <ul style="list-style-type: none"> <li>Represent linear relationships in various forms, including tables, graphs, equations, expressions, and inequalities.</li> <li>Use properties of operations to generate equivalent expressions.</li> <li>Write, read, and evaluate expressions in which letters stand for numbers</li> </ul> | <ul style="list-style-type: none"> <li>Solve real-life and mathematical problems using numerical and algebraic equations</li> <li>Graph solutions and solution sets of equations and inequalities, and interpret them in the context of the problems</li> </ul>  |
| Possible tasks*   | MAP Task A05 “Baseball Jersey’s”; CMI 2 8 <sup>th</sup> Shapes of Algebra: Problem 5, 2; Illustrative Mathematics: “Shrinking”; and “Art Class”  | *The Grade 7 mathematics toolset includes this and other resources and can be found online at <a href="https://ocs.cps.k12.il.us/sites/IKMC/default.aspx">https://ocs.cps.k12.il.us/sites/IKMC/default.aspx</a> and on the Department of Mathematics and Science website ( <a href="http://cmsi.cps.k12.il.us">cmsi.cps.k12.il.us</a> ). |

# Grade 7 Mathematics Planning Guide – SY12-13

## Big Idea: Expressions and Equations

### Topic: Analyzing Linear Relationships

#### CCSS-M Content Standards

- 7.EE.4a Solve word problems leading to equations of the form  $px + q = r$  and  $p(x + q) = r$  where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Solve equations of these forms fluently.  
Compare and algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.
- 7.EE.4b Solve word problems leading to inequalities of the form  $px + q > r$  or  $px + q < r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Graph the solution set of inequalities and interpret them in the contexts of the problems.
- 7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

| Connections to Standards for Mathematical Practice       | Standards for Mathematical Practice   | How it applies...   |
|--|---|---|
|  | MP1 - Make sense of the problem and persevere in solving it.  | Through defining what the word problems are asking (7.EE.4a/b); comparing an algebraic solution to an arithmetic solution (7.EE.4a) and assessing the reasonableness of an answer (7.EE.3)  |
|  | MP2 - Reason abstractly and quantitatively.   | Decontextualizing - creating an equation or expression that gets transformed and then contextualizing - connecting the new expression or equation back into the context of the problem  |
|  | MP4 – Modeling with mathematics.  | Creating an equation from the context of a word problem.  |
| Key Ideas and Terms for “Analyzing Linear Relationships” | Key Ideas   | Key Terms   |
|  | <ul style="list-style-type: none"><li>Analyze and interpret single variable expressions, equations, inequalities, tables, and graphs to solve problems in context.</li></ul>  | Expression, equation, inequality, rational numbers, solution<br><i>Terms should be deeply understood within the context of their use. Not to be considered standalone vocabulary exercises.</i>   |
| Prior Knowledge  | <ul style="list-style-type: none"><li>Knowledge of and capability to plot points in all four quadrants of the coordinate plane</li><li>Understand rates in the forms of equivalent ratios and tables</li><li>Write, read, and evaluate expressions in which letters stand for numbers</li></ul> | <ul style="list-style-type: none"><li>Identify when two expressions are equivalent</li><li>Understand that positive and negative numbers are used together to describe quantities having opposite directions or values</li></ul>  |
| Students will be able to:                                | <ul style="list-style-type: none"><li>Solve linear equations in one variable.</li><li>Identify linear equations with one, infinitely many, or no solution.</li><li>Transform an expression or equation into alternate forms.</li></ul>  | <ul style="list-style-type: none"><li>Find the solution to a system of equations from a graph.</li><li>Solve systems algebraically using substitution and elimination/combination.</li><li>Understand the solution of a system of equations in the context of the problem and write as an ordered pair.</li></ul> |
| Possible tasks*  | Inside Mathematics Problems of the Month: “Growing Staircases” and “Squirreling it Away”; CPM – Engaging Inequalities   |   |

\*The Grade 7 mathematics toolset includes this and other resources and can be found online at <https://ocs.cps.k12.il.us/sites/IKMC/default.aspx> and on the Department of Mathematics and Science website ([cmsi.cps.k12.il.us](http://cmsi.cps.k12.il.us)).

# Grade 8 Mathematics Planning Guide – SY12-13

## Introduction

The purpose of this Planning Guide is to define the scope of the Common Core State Standards for Mathematics (CCSS-M) Content Standards to be taught in the 2012-2013 school year. It has been designed by CPS teachers to be useful to CPS teachers during the three-year transition to full implementation of the CCSS-M.

In Year 1 of the transition to CCSS-M, 2012-2013, grades 6-8 mathematics teachers will be teaching:

- **To the former ILS before the ISAT in March.** The expectation is for schools and teachers to continue using their current mathematics instructional materials and to address the Illinois Assessment Frameworks from the beginning of the year until the ISAT is administered. In addition to this content, teachers will explicitly integrate the Standards for Mathematical Practice into their lessons on a regular basis.
- **To the new CCSS-M standards, per the scope described by this Planning Guide, after the ISAT,** using their instructional materials and other resources to integrate mathematical practices with rigorous mathematical tasks that support the scope of content standards. A toolset to support this instruction includes (1) samples of rigorous grade-specific tasks (including samples of formative assessment options based on MARS (Mathematics Assessment Resource Service) tasks) and (2) general tools that include a sample lesson planning template, a tool for analyzing and modifying lessons/tasks; samples of modified lessons/tasks; and a list of professional resources. These tools are available at <https://ocs.cps.k12.il.us/sites/IKMC/default.aspx> and on the Department of Mathematics and Science website (<http://cmsi.cps.k12.il.us>).

Specifically, after the March ISAT, grades 6-8 mathematics teachers will focus on the *Expressions and Equations* progression, as defined in the Progressions for the *Common Core State Standards in Mathematics: 6-8, Expressions and Equations* (2011). This progression was chosen because the Grade 8 CCSS-M includes the “algebra of lines,” which was formerly taught in the first half of high school Algebra I courses (“algebra of lines” refers to equations, graphs of linear relationships, and systems of linear equations). Concentrating on this particular progression allows grades 6-8 classrooms to hone in on the vertical articulation of this very important concept, focusing on students’ cognitive development and the logical structure of the CCSS-M.

Please note: If the content standards in this Planning Guide are covered within the scope of a classroom’s instructional materials prior to ISAT, teachers should teach to these with the level of rigor expected by the CCSS-M, integrating the Standards for Mathematical Practice with the content. Post-ISAT is an opportunity to reinforce content standards already addressed, and to focus more deeply on the *Expressions and Equations* content standards that were not covered prior to ISAT.

During the first 3 quarters, teachers will explicitly integrate the Standards for Mathematical Practice (below) into their lessons on a regular basis. In the 4th quarter, instruction that integrates Standards for Mathematical Practice with targeted content standards is supported by “how to” guidance in this Planning Guide. By teaching the mathematical practices alongside the indicated content standards, students will be more likely to achieve the depth of conceptual understanding and procedural fluency expected by the CCSS-M.

## The CCSS-M Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Following the ISAT, this Planning Guide will form the basis for instruction in Grade 8 mathematics classrooms. This year's focus of learning, *Expressions and Equations*, is broken into three topics: 1) What is a Function? 2) Linear Functions and Equations and 3) Solving Linear Equations and Systems. Topic components include a targeted set of content standards, target mathematical practices (and how they apply to the specific topic), key ideas for learning, key terms, and sample instructional tasks.

The guide assumes 9 weeks for instruction, including time for formative and summative assessments. The topics are sequenced in a way that we believe best develops and connects the mathematical content of the CCSS-M. However, teachers should review the topics and decide the order and time allocation appropriate for their classrooms, given their students, instructional materials, and other considerations. The order of the standards included in a topic does not imply a sequence of the content. Some standards may be revisited several times while addressing the topic, while others may be only partially addressed, depending on the mathematical focus of the topic.

Finally, this document reflects our current thinking about the transition to the CCSS-M. We welcome feedback about your experience with the document. Please share your thoughts with your network staff who will forward to the Department of Mathematics and Science.

# Grade 8 Mathematics Planning Guide – SY12-13

## Big Idea: Expressions and Equations

### Big Idea Assessment: Expressions and Equations

See the 8<sup>th</sup> Grade Toolset\*: MARS task: Sorting Functions, Summative for this Big Idea

#### Topic: What is a Function?

##### CCSS-M Content Standards

- 8.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.

8.F.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

- 8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

| Connections to Standards for Mathematical Practice | Standards for Mathematical Practice   | How it applies...   |
|--|---|---|
|  | MP7 - Look for and make use of structure.   | How different components of an equation affect or are represented in the graph of the equation; how components of the graph can lead to developing an equation  |
|  | MP8 - Look for and express regularity in repeated reasoning.  | Through drawing the graphs of various equations, students will recognize the effect of different portions of the equation on the graphs; through repeating calculations, students express the generalized set of operations and create an equation.   |
| Key Ideas and Terms for "What is a Function?"      | Key Ideas   | Key Terms   |
|  | <ul style="list-style-type: none"><li>Definition and characteristics of different types of functions</li><li>Relationship between graph and equation</li><li>Development and consistent use of the language of functions</li></ul>  | <p>Function, input, output, independent variable, dependent variable, rate of change, Cartesian plane, slope, intercept, proportional</p> <p>Terms should be deeply understood within the context of their use. Not to be considered standalone vocabulary exercises.</p>   |
| Prior Knowledge                                    | <ul style="list-style-type: none"><li>Use of letters as variables rather than boxes, etc. as in earlier grades</li><li>Familiarity with graphing in all four quadrants of the Cartesian plane</li><li>Understanding of the meaning of operations, particularly with integers</li></ul>  |   |
| Students will be able to:                          | <ul style="list-style-type: none"><li>Move between tables of value and function rules to represent functions.</li><li>Graph functions using input-output values as ordered pairs and identify type of function through the shape of the graph.</li><li>Match corresponding tables, graphs, equations.</li><li>Draw a graph given verbal descriptions.</li></ul> | <ul style="list-style-type: none"><li>Identify shapes of graphs of parent functions (linear, quadratic, cubic, exponential, etc.) and the effect of different components of the equation (leading coefficient, constant term, etc.).</li><li>Graph proportional relationships.</li><li>Compare different proportional relationships represented in different forms.</li></ul> |
| Possible Task(s)*                                  | CME Algebra 1 text, Chapter 3, for shapes of graphs of parent functions   |   |

\*The Grade 8 mathematics toolset includes this and other resources and can be found online at <https://ocs.cps.k12.il.us/sites/IKMC/default.aspx> and on the Department of Mathematics and Science website ([cmsi.cps.k12.il.us](http://cmsi.cps.k12.il.us)).

# Grade 8 Mathematics Planning Guide – SY12-13

## Big Idea: Expressions and Equations

### Topic: Linear Functions and Equations

#### CCSS-M Content Standards

- 8.EE.6 Use similar triangles to explain why the slope  $m$  is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation  $y = mx$  for a line through the origin and the equation  $y = mx + b$  for a line intercepting the vertical axis at  $b$ .  
 Interpret the equation  $y = mx + b$  as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.
- 8.F.3 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two  $(x, y)$  values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
- 8.SP.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.

| Connections to Standards for Mathematical Practice       | Standards for Mathematical Practice  | How it applies...  |
|--|--|--|
|  | MP2 - Reasoning abstractly and quantitatively.   | Decontextualizing is working strictly with the equation and the graph; contextualizing is interpreting the slope as the rate of change and the $y$ -intercept as the initial value of the context or situation. This also involves understanding when a graph should be continuous or when it will not continue from the first quadrant to the second, third, or fourth. |
|  | MP7- Look for and make use of structure.   | How different components of an equation affect or are represented in the graph of the equation or arise from context.  |
|  | MP8 - Look for and express regularity in repeated reasoning.   | Repeated calculation of slope leads to an equation for the graph of a line or a linear context.  |
| Key Ideas and Terms for “Linear Functions and Equations” | Key Ideas  | Key Terms  |
|  | <ul style="list-style-type: none"> <li>• Equations and graphs of lines</li> <li>• Building linear equations from a context</li> </ul>  | <p>Slope, intercepts, slope-intercept form, proportional terms should be deeply understood within the context of their use. Not to be considered standalone vocabulary exercises.</p>  |
| Prior Knowledge  | <ul style="list-style-type: none"> <li>• Fluency with arithmetic operations and the connections to real world contexts</li> <li>• Exposure to proportional relationships and reasoning</li> <li>• Understanding that a graph is a picture of the set of coordinate pairs that satisfy an equation</li> </ul>   |  |
|  | <ul style="list-style-type: none"> <li>• Translate from English to algebraic expressions/equations.</li> <li>• Use similar triangles to determine why slope is same between collinear points.</li> <li>• Derive a linear equation from a context and transform it into <math>y=mx</math> or <math>y=mx+b</math> and interpret the meaning of each component of the equation.</li> <li>• Recognize non-linear equations.</li> </ul> | <ul style="list-style-type: none"> <li>• Determine rate of change in linear relationships.</li> <li>• Move fluently between a table, a graph and an equation.</li> <li>• Describe the relationship between two variables.</li> <li>• Interpret meaning of slope and <math>y</math>-intercept in contexts.</li> </ul>   |
| Students will be able to:                                |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

Possible task(s)\*

Triangle Task (NYC EE Unit); CME lesson 4.1; Aussie Fir Trees (NYC EE unit); Square Patterns (MARS); EZ Coasters (NYC EE Unit)

Two Oil Tanks (NYC EE Unit) Summative for this topic and slight bridge to systems

\* The Grade 8 mathematics toolset includes this and other resources and can be found online at <https://ocs.cps.k12.il.us/sites/IKMC/default.aspx> and on the Department of Mathematics and Science website ([cmci.cps.k12.il.us](http://cmci.cps.k12.il.us)).

# Grade 8 Mathematics Planning Guide – SY12-13

## Big Idea: Expressions and Equations

### Topic: Solving Linear Equations and Systems CCSS-M Content Standards

|  |   |   |   |  |  |  |  |
|--|---|---|---|--|--|--|--|
| 8.EE.7   | Solve linear equations in one variable. <ol style="list-style-type: none"><li>Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form <math>x = a</math>, <math>a = a</math>, or <math>a = b</math> results (where <math>a</math> and <math>b</math> are different numbers).</li><li>Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.</li></ol> | 8.EE.8  | Analyze and solve pairs of simultaneous linear equations. <ol style="list-style-type: none"><li>Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.</li><li>Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.</li><li>Solve real-world and mathematical problems leading to two linear equations in two variables.</li></ol> | Connections to Standards for Mathematical Practice | Standards for Mathematical Practice  | How it applies...<br><br>MP4 - Model with mathematics.<br><br>MP7- Look for and make use of structure. | How it applies...<br><br>Creating an equation that represents the situation and captures the generalized situation of the context.<br><br>How different components of an equation affect or are represented in the graph of the equation or arise from context and use various representations of linear relationships and identify benefits to using each form. |
| Key Ideas and Terms for “Solving Linear Equations and Systems” | Key Ideas   | Key Terms   | Break-even point, intersection point, substitution, elimination, like terms, equivalence, distributive property, solution<br><br><i>Terms should be deeply understood within the context of their use. Not to be considered standalone vocabulary exercises.</i>  | Prior Knowledge                                    | Substituting values for variables to evaluate expressions (value of $3x + 4$ if $x=8$ )<br><br>Apply the distributive property | Understand equivalent expressions<br><br>Combining like terms  | Find the solution to a system of equations from a graph.<br><br>Solve systems algebraically using substitution and elimination/<br>combination.<br><br>Understand the solution of a system of equations in the context of the problem and write as an ordered pair.  |
| Students will be able to:                                      | Solve linear equations in one variable.<br><br>Identify linear equations with one, infinitely many, or no solution.<br><br>Transform an expression or equation into alternate forms.  | Carnegie car rentals from two companies; Cell Phone Plans | CME lessons 4-10, 4-12; Carnegie car rentals from two companies; Cell Phone Plans   |  |  |  |  |
| Possible task(s)*  |   |   | *The Grade 8 mathematics toolset includes this and other resources and can be found online at <a href="https://ocs.cps.k12.il.us/sites/IKMC/default.aspx">https://ocs.cps.k12.il.us/sites/IKMC/default.aspx</a> and on the Department of Mathematics and Science website ( <a href="http://cmsi.cps.k12.il.us">cmsi.cps.k12.il.us</a> ).  |  |  |  |  |

# Algebra I Planning Guide – SY12-13

## Introduction

The purpose of this Algebra I Planning Guide is to define the scope of the Common Core State Standards for Mathematics (CCSS-M) content standards to be taught in the 2012-2013 school year. It has been designed by CPS teachers to be useful to CPS teachers during the three-year transition to full implementation of the CCSS-M.

Algebra I typically extends the mathematics that students learn in the middle grade courses. However, in CPS, prior to CCSS-M, “algebra of lines” has not been fully introduced in the middle grades. (“Algebra of lines” refers to equations, graphs of linear relationships, and systems of linear equations.) Therefore, the Algebra I Planning Guide (this document) includes this concept. To accommodate the time required to teach “algebra of lines,” the guide does not include the Statistics and Probability content outlined in the traditional Algebra I course in the *Appendix A: Designing High School Mathematics Courses on the Common Core State Standards* (2011). This scope of CCSS-M content standards for this first year of transition will still be rather ambitious for Algebra I.

This Planning Guide is structured around six Big Ideas (below). For each Big Idea, a summative assessment is referenced, and topics provide the detailed components to support instruction. Topic components include a targeted set of content standards, target mathematical practices (and how they apply to the specific topic), and sample instructional tasks. The sample tasks have been selected for their rigor and their potential usefulness as formative assessments for the topic. A toolset to support instruction includes: a lesson planning template; samples of rigorous tasks; samples of formative assessment options based on MARS (Mathematics Assessment Resource Service) tasks; tools for examining and modifying lessons/tasks to increase rigor; samples of modified lessons/tasks; and a recommended list of professional resources. These tools are available at <https://ocs.cps.k12.il.us/sites/IKMC/default.aspx> and on the Department of Mathematics and Science website (<http://cmsi.cps.k12.il.us>).

The guide assumes 159 days for instruction, including time for formative and summative assessments. The Big Ideas are sequenced in a way that we believe best develops and connects the mathematical content of the CCSS-M. However, teachers should review the Big Ideas and decide the order and time allocation appropriate for their classrooms, given their students, instructional materials, and other considerations. The order of the standards presented in a topic **does not imply a sequence of the content**. Some standards may be revisited several times while addressing the topic, while others may be only partially addressed, depending on the topic’s mathematical focus.

Throughout Algebra I, students should continue to develop proficiency in the Common Core’s Standards for Mathematical Practice (below). Teachers should integrate the instruction of content standards with mathematical practices. This Planning Guide includes “how to” guidance to support this approach to integrated instruction. When the mathematical practices are taught alongside the indicated content standards, students will be more likely to achieve the depth of conceptual understanding and procedural fluency that are expected by the CCSS-M.

## The CCSS-M Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning

Finally, this document reflects our current thinking about the transition to the CCSS-M. We welcome feedback about your experience with the document. Please share your thoughts with your network staff who will forward to the Department of Mathematics and Science.

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# Algebra I Planning Guide – SY12-13

## Big Idea: Foundations for Algebra

See Algebra I Toolset\* : Central Park

### Big Idea Assessment: Foundations for Algebra

#### Topic: Operations on Rational Numbers

##### CCSS-M Content Standards

- 6-NS.7c Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or a negative quantity in a real-world situation.
- 7-NS.1b Understand  $p + q$  as the number located a distance of  $|q|$  from  $p$ , in the positive or negative direction depending on whether  $q$  is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
- 7-NS.1c Understand that subtraction of rational numbers as adding the additive inverse,  $p - q = p + (-q)$ . Show that the distance of two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
- 7-NS.1d Apply properties of operations as strategies to add and subtract rational numbers.
- 7-NS.2a Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as  $(-1)(-1) = 1$  and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

| Connections to Standards for Mathematical Practice | Standards of Mathematical Practice  | How it applies...   |
|--|---|---|
|  | MP2 - Reason abstractly and quantitatively.   | Manipulate the mathematical representation by showing the process considering the meaning of the quantities involved.   |
|  | MP3 - Construct viable arguments and critique the reasoning of others.  | Justify (orally and in written form) the approach used, including how it fits in the context from which the data arose.   |
|  | MP7 - Look for and make use of structure.   | Use patterns or structure to make sense of mathematics and connect prior knowledge to similar situations and extend to novel situations.  |
| Prior Knowledge                                    | <ul style="list-style-type: none"><li>• The number line</li><li>• Rational numbers</li><li>• Additive inverse</li></ul>   | <ul style="list-style-type: none"><li>• Properties of operations</li><li>• Properties of operations, esp. the distributive property</li></ul>   |
| Students will be able to:                          | <ul style="list-style-type: none"><li>• Find the absolute value of a rational number.</li><li>• Explain the meaning of absolute value as its distance from 0 on the number line.</li><li>• Add rational numbers.</li><li>• Use the number line and the definition of absolute value to explain how to add rational numbers.</li><li>• Explain why the sum of two opposite numbers is 0.</li><li>• Apply the skill of adding rational numbers to the real world.</li></ul> | <ul style="list-style-type: none"><li>• Subtract rational numbers.</li><li>• Use the concepts of absolute value and additive inverse to explain how to subtract rational numbers.</li><li>• Apply the skill of subtracting rational numbers to the real world.</li><li>• Use properties of operations when adding and subtracting rational numbers.</li><li>• Multiply rational numbers.</li><li>• Interpret products of rational numbers in real world contexts.</li></ul> |
| Possible task(s)*                                  |   |   |

\* The Algebra I toolset includes this and other resources and can be found online at <https://ocs.cps.k12.il.us/sites/IKVIC/default.aspx> and on the Department of Mathematics and Science website ([cmst.cps.k12.il.us](http://cmst.cps.k12.il.us)).

# Algebra I Planning Guide – SY12-13

## Big Idea: Foundations for Algebra

### Topic: Working with Expressions

#### CCSS-M Content Standards

- 6-EE.2c Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).
- 6-EE.3 Apply the properties of operations to generate equivalent expressions.
- 6-EE.4 Identify when two expressions name the same number regardless of which value is substituted into them.
- 7-EE.1 Apply properties of operations as strategies to add, subtract, factor and expand linear expressions with rational coefficients.

| Connections to Standards for Mathematical Practice | Standards of Mathematical Practice  | How it applies...  |
|--|---|--|
|  | MP1 - Make sense of problems and persevere in solving them.   | Evaluate progress toward the solution and make revisions if necessary.   |
|  | MP2 - Reason abstractly and quantitatively.   | Recognize the relationships between numbers/quantities within the process to evaluate a problem.   |
|  | MP6 - Attend to precision.  | Calculate answers efficiently and accurately and label them appropriately.   |
| Prior Knowledge                                    | <ul style="list-style-type: none"><li>• Order of operations</li><li>• Meaning of exponents</li><li>• Four operations on rational numbers</li></ul>  | <ul style="list-style-type: none"><li>• Properties of operations</li><li>• Meaning of “equivalent”</li><li>• Evaluating expressions</li></ul>                  |
| Students will be able to:                          | <ul style="list-style-type: none"><li>• Evaluate algebraic expressions, including those with exponents and those requiring knowledge of order of operations.</li><li>• Apply the properties of operations to generate equivalent expressions.</li></ul> | <ul style="list-style-type: none"><li>• Recognize equivalent expressions.</li><li>• Use properties of operations to generate equivalent expressions.</li></ul> |
| Possible task(s)*                                  | A Million Dollars   | From: Mathematics Assessment Project   |

\*The Algebra I toolset includes this and other resources and can be found online at <https://ocs.cps.k12.il.us/sites/IKM/C/default.aspx> and on the Department of Mathematics and Science website ([cmstools.cps.k12.il.us](http://cmstools.cps.k12.il.us)).

# Algebra I Planning Guide – SY12-13

## Big Idea: Foundations for Algebra

### Topic: Introduction to Equations and Functions

#### CCSS-M Content Standards

|         |  |  |
|---------|--|--|
| 7-EE.4  | Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.  |  |
| A-CED.2 | Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.  |  |
| 6-EE.9  | Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. |  |
| 7-RP.2a | Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.   |  |
| 7-RP.2b | Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.  |  |
| 7-RP.2d | Explain what a point $(x, y)$ on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where $r$ is the unit rate.  |  |
| 8-EE.5  | Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.  |  |
| F-IF.1  | Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$ . The graph of $f$ is the graph of the equation $y = f(x)$ .  |  |
| F-IF.2  | Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.  |  |
| F-IF.5  | Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. <i>For example, if the function <math>h(n)</math> gives the number of person-hours it takes to assemble <math>n</math> engines in a factory, then the positive integers would be an appropriate domain for the function.★</i>   |  |

★ Specific modeling standard (versus an example of the modeling Standard for Mathematical Practice)

| Connections to Standards for Mathematical Practice | Standards of Mathematical Practice                                     | How it applies...   |
|--|--|---|
|  | MP2 - Reason abstractly and quantitatively.                            | Evaluate progress toward the solution and make revisions if necessary.  |
|  | MP3 - Construct viable arguments and critique the reasoning of others. | Use observations and prior knowledge (stated assumptions, definitions, and previous established results) to make conjectures and construct arguments.   |
|  | MP6 - Attend to precision.   | Calculate answers efficiently and accurately and label them appropriately.  |
| Prior Knowledge                                    |  | <ul style="list-style-type: none"><li>• Concept of "unit rate" or slope</li><li>• Meaning of "variable"</li><li>• Meaning of equal and inequality signs</li><li>• Translating words into symbols</li><li>• Reasoning from words to symbols</li><li>• Meaning of dependent and independent variables</li><li>• Meaning of "proportional"</li><li>• Equivalent ratios</li><li>• Graphing linear relationships</li></ul> |

# Algebra I Planning Guide – SY12-13

## Big Idea: Foundations for Algebra

|                           |   |
|---------------------------|---|
| Students will be able to: | <ul style="list-style-type: none"><li>• Create equations or inequalities to describe situations in real-world or mathematical problems.</li><li>• Write equations in two variables to describe real world situations.</li><li>• Graph a relationship between two variables in the coordinate plane.</li><li>• Analyze the relationship between the two variables using both tables and graphs.</li><li>• Use graphing calculator to analyze relationship.</li><li>• Decide whether a relationship between two quantities is proportional by<ul style="list-style-type: none"><li>a) Using ratios</li><li>b) Analyzing the graph of the relationship</li></ul></li><li>• Identify the constant of proportionality in a proportional relationship between two variables by examining:<ul style="list-style-type: none"><li>a) Tables</li><li>b) Graphs</li><li>c) Equations</li><li>d) Diagrams</li><li>e) Verbal description</li></ul></li></ul> |
| Possible task(s)*         | The Wheel Shop  |

From: Inside Mathematics

\*The Algebra I toolset includes this and other resources and can be found online at <https://ocs.cps.k12.il.us/sites/IMYC/default.aspx> and on the Department of Mathematics and Science website ([cmst.cps.k12.il.us](http://cmst.cps.k12.il.us)).

# Algebra I Planning Guide – SY12-13

## Big Idea: Introduction to Functions and Their Rules

**Big Idea Assessment: Introduction to Functions and Their Rules**  
See Algebra I Toolset\*: Cell Phones (from Illustrative Mathematics)

### Topic: Representing Mathematical Relationships in Multiple Ways

#### CCSS-M Content Standards

- A-CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.  
N-Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
- F-IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- 6-EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.

| Connections to Standards for Mathematical Practice | Standards of Mathematical Practice  | How it applies...   |
|--|---|---|
|  | MP1 - Make sense of problems and persevere in solving them.   | Analyze given information to develop possible strategies for solving the problem.   |
|  | MP2 - Reason abstractly and quantitatively.   | Manipulate the mathematical representation by showing the process considering the meaning of the quantities involved.   |
|  | MP7 - Look for and make use of structure.   | Use patterns or structure to make sense of mathematics and connect prior knowledge to similar situations and extend to novel situations.  |
|  | MP8 - Look for and express regularity in repeated reasoning.  | Generalize the process to create a shortcut which may lead to developing rules or creating a formula.   |
| Prior Knowledge                                    | <ul style="list-style-type: none"><li>• Use variables to represent numbers.</li><li>• Write expressions when solving a real world or mathematical problem.</li></ul>  | <ul style="list-style-type: none"><li>• Solve equations.</li><li>• Apply properties of operations.</li></ul>  |
| Students will be able to:                          | <ul style="list-style-type: none"><li>• Solve real-world problems and model real-world situations using patterns and mathematical relationships.</li><li>• Make connections among representations of mathematical relationships, using words, pictures, tables, graphs, and algebraic rules.</li><li>• Compare and contrast to determine the advantages and limitations of using a particular representation to answer a question.</li><li>• Analyze and create equivalent algebraic expressions and rules.</li></ul> | <ul style="list-style-type: none"><li>• Plot points and scale axes.</li><li>• Keep track of steps or processes to reflect and observe patterns that will aid in the formulation of equations that arise from functions.</li><li>• Understand the concept of a functional relationship as well as the basic aspects of linear relationships.</li><li>• Demonstrate an understanding of “reasonable inputs” and discrete and continuous data.</li></ul> |
| Possible task(s)*                                  | Printing Tickets  | From: Mathematics Assessment Project  |

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# Algebra I Planning Guide – SY12-13

## Big Idea: Introduction to Functions and Their Rules

### Topic: Further Examination of Functions and Equations

#### CCSS-M Content Standards

|         |  |
|---------|--|
| F-IF.2  | Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.  |
| F-IF.5  | Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.<br><i>For example, if the function <math>n(h)</math> gives the number of person-hours it takes to assemble <math>n</math> engines in a factory, then the positive integers would be an appropriate domain for the function.</i> |
| 7.RP.2a | Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.   |
| 7.RP.2b | Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.  |
| 7.RP.2d | Explain what a point $(x, y)$ on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where $r$ is the unit rate.  |
| 8.EE.5  | Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.  |

| Connections to Standards for Mathematical Practice | Standards of Mathematical Practice   | How it applies...   |
|--|--|---|
|  | MP3 - Construct viable arguments and critique the reasoning of others.   | Use observations and prior knowledge (stated assumptions, definitions, and previous established results) to make conjectures and construct arguments.   |
|  | MP4 - Model with mathematics.  | Use a variety of methods to model, represent, and solve real-world problems.  |
|  | MP5 - Use appropriate tools strategically.   | Select and use appropriate tools to best model/solve problems.  |
|  | MP6 - Attend to precision.   | Calculate answers efficiently and accurately and label them appropriately.  |
| Prior Knowledge                                    | Represent proportional relationships by equations.   |   |
| Students will be able to:                          | <ul style="list-style-type: none"><li>• Represent functions using words, tables, graphs, and symbols.</li><li>• Identify independent variables in functional relationships.</li><li>• Use function notation.</li></ul> | <ul style="list-style-type: none"><li>• Recognize difference between proportional and non-proportional situations represented by linear functions.</li><li>• Determine the constant in direct variation situations.</li></ul> |
| Possible task(s)*                                  | A Golden Crown   | From: Mathematics Assessment Project  |

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# Algebra I Planning Guide – SY12-13

## Big Idea: Linear Equations and Inequalities

### Big Idea Assessment: Linear Equations and Inequalities See Algebra I Toolset\*: The Road Trip!

#### Topic: Linear Equations and Inequalities

##### CCSS-M Content Standards

- 7-EE.4a Solve word problems leading to equations of the form  $px + q = r$  and  $p(x + q) = r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.
- 8-EE.7a Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form  $x = a$ ,  $a = a$ , or  $a = b$  results (where  $a$  and  $b$  are different numbers).
- 8-EE.7b Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

- A-CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law  $V = I/R$  to highlight resistance  $R$ . A-REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

- A-REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
- A-REI.11 Explain why the  $x$ -coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = g(x)$  intersect are the solutions of the equation  $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where  $f(x)$  and/or  $g(x)$  are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
- F-LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).

- 7-EE.4b Solve word problems leading to inequalities of the form  $px + q > r$  or  $px + q < r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.

- A-CED.3 Represent constraints by equations or inequalities and/or by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

- A-REI.3 Solve linear equations and inequalities in one variable, including equations and coefficients represented by letters.

- A-REI.12 Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

#### Connections to Standards for Mathematical Practice

| Standards of Mathematical Practice          | How it applies...  |
|---|--|
| MP2 - Reason abstractly and quantitatively. | Translate given information to create a mathematical representation for a concept.   |
| MP4 - Model with mathematics.               | Choose a model that is both appropriate and efficient to arrive at one or more desired solutions.  |
| MP7 - Look for and make use of structure.   | Use patterns or structure to make sense of mathematics and connect prior knowledge to similar situations and extend to novel situations. |

#### Prior Knowledge

- Use variables to represent numbers
- Write expressions when solving a real world or mathematical problem
- Solve equations
- Apply properties of operations

# Algebra I Planning Guide – SY12-13

## Big Idea: Linear Equations and Inequalities

|                           |   |   |
|---------------------------|---|---|
| Students will be able to: | <ul style="list-style-type: none"><li>• Communicate mathematical ideas and conclusions through language and representation.</li><li>• Use reasoning to make conjectures and verify conclusions.</li><li>• Analyze situations involving linear functions and formulate linear equations to solve problems.</li><li>• Use different methods for solving linear equations: using concrete models, graphs, and the properties of equality.</li><li>• Choose an appropriate method, and solve the equations.</li><li>• Apply techniques for solving equations in one variable to solve literal equations.</li><li>• Compare and contrast to determine the advantages and limitations of using a particular representation to answer a question.</li><li>• Analyze and create equivalent algebraic expressions and rules.</li></ul> | <ul style="list-style-type: none"><li>• Write inequalities in one and two variables to represent problem situations.</li><li>• Solve linear inequalities in one variable using tables, graphs, and algebraic operations.</li><li>• Solve a linear system of equations with two variables.</li><li>• Graph solutions to linear inequalities in one variable on a number line.</li><li>• Graph solutions to linear inequalities in two variables on a coordinate plane.</li><li>• Graph solutions to systems of linear inequalities in two variables on a coordinate plane.</li></ul> |
|---------------------------|---|---|

Possible task(s)\*

From: Mathematics Assessment Project

\*The Algebra I toolset includes this and other resources and can be found online at <https://oer.cps.k12.il.us/sites/KM/C/default.aspx> and on the Department of Mathematics and Science website ([cnsl.cps.k12.il.us](http://cnsl.cps.k12.il.us)).

# Algebra I Planning Guide – SY12-13

## Big Idea: Modeling with Linear Functions

### Big Idea Assessment: Modeling with Linear Functions See Algebra I Toolset\*: Buying Chips and Candy

#### Topic: Linearity as Constant Rate of Change

##### CCSS-M Content Standards

- 8-F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear), Sketch a graph that exhibits the qualitative features of a function that has been described verbally (e.g., graphing a distance v. time story).
- F-IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- F-LE.1.b Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
- 7-RP.2a Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
- 7-RP.2b Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

| Connections to Standards for Mathematical Practice | Standards of Mathematical Practice   | How it applies...  |
|--|--|--|
|  | MP2 - Reason abstractly and quantitatively.  | Recognize the relationships between numbers/quantities within the process to evaluate a problem.   |
|  | MP3 - Construct viable arguments and critique the reasoning of others.   | Justify (orally and in written form) the approach used, including how it fits in the context from which the data arose.  |
|  | MP4 - Model with mathematics.  | Simplify a complicated problem by making assumptions and approximations.   |
|  | MP8 - Look for and express regularity in repeated reasoning.   | Recognize similarities and patterns in repeated trials with a process.   |
| Prior Knowledge                                    | <ul style="list-style-type: none"><li>Interpret unit rate as the slope.</li><li>Understand ratios.</li></ul>   | <ul style="list-style-type: none"><li>Compute unit rates associated with ratios of fractions.</li></ul>  |
| Students will be able to:                          | <ul style="list-style-type: none"><li>Show understanding of the concepts of speed and rate.</li><li>Create motion graphs (distance vs. time).</li><li>Describe how changes in motion affect the graph.</li></ul> | <ul style="list-style-type: none"><li>Use common or first differences to determine if a function is linear.</li><li>Identify relationships as linear or nonlinear using a table, graph, or equation.</li><li>Find rates for data in tables, graphs and problem situations.</li></ul> |
| Possible task(s)*                                  |  |  |

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# Algebra I Planning Guide – SY12-13

## Big Idea: Modeling with Linear Functions

### Topic: Constructing Linear Functions

#### CCSS-M Content Standards

- F-LE.2 Construct linear ... functions, including arithmetic ... sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).  
A-CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on a coordinate axes with labels and scales.  
8-F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two ( $x, y$ ) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
- F-IF.7a Graph linear ... functions and show intercepts...

| Connections to Standards for Mathematical Practice | Standards of Mathematical Practice   | How it applies...   |
|--|--|---|
|  | MP2 - Reason abstractly and quantitatively.  | Translate given information to create a mathematical representation for a concept.  |
|  | MP7 - Look for and make use of structure.  | Use patterns or structure to make sense of mathematics and connect prior knowledge to similar situations and extend to novel situations.  |
|  | MP8 - Look for and express regularity in repeated reasoning.   | Evaluate the reasonableness of results throughout the mathematical process while attending to detail.   |
| Prior Knowledge                                    | <ul style="list-style-type: none"><li>• Apply properties of operations.</li><li>• Create equations and inequalities in one variable.</li></ul>   |   |
| Students will be able to:                          | <ul style="list-style-type: none"><li>• Write an equation based on an arithmetic sequence.</li><li>• Determine a recursive formula.</li><li>• Explain how to determine the next step in a pattern.</li></ul> | <ul style="list-style-type: none"><li>• Know and use the relationship between the <math>y</math>-intercept of the graph of a linear model and the situation being modeled.</li><li>• Use constant rate of change and slope to analyze and graph linear functions.</li></ul> |
| Possible task(s)*                                  |  |   |

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# Algebra I Planning Guide – SY12-13

## Big Idea: Modeling with Linear Functions

### Topic: Analyzing Linear Functions

#### CCSS-M Content Standards

- F-IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
- F-BF.3 Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
- F-LE.5 Interpret the parameters in a linear or exponential function in terms of a context.
- 8.F.3 Interpret the equation  $y = mx + b$  as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.

| Connections to Standards for Mathematical Practice | Standards of Mathematical Practice  | How it applies...  |
|--|---|--|
|  | MP2 - Reason abstractly and quantitatively.   | Translate given information to create a mathematical representation for a concept.   |
|  | MP3 - Construct viable arguments and critique the reasoning of others.  | Justify (orally and in written form) the approach used, including how it fits in the context from which the data arose.  |
|  | MP4 - Model with mathematics.   | Simplify a complicated problem by making assumptions and approximations.   |
|  | MP8 - Look for and express regularity in repeated reasoning.  | Evaluate the reasonableness of results throughout the mathematical process while attending to detail.  |
| Prior Knowledge                                    | <ul style="list-style-type: none"><li>Interpret unit rate as the slope.</li><li>Understand ratios.</li></ul>  | <ul style="list-style-type: none"><li>Compute unit rates associated with ratios of fractions.</li></ul>  |
| Students will be able to:                          | <ul style="list-style-type: none"><li>Write the equation of a line in different forms (slope-intercept, standard, and point-slope forms).</li><li>Identify slope and y-intercept from graphs, tables and problem situations.</li><li>Identify equations as linear or non-linear</li></ul> | <ul style="list-style-type: none"><li>Understand the effects of changing <math>m</math> or <math>b</math> on the graph of <math>y = mx + b</math>.</li><li>Transform the parent function <math>y = x</math> to create other linear functions.</li><li>Interpret the meaning of <math>m</math> and <math>b</math> from tables and graphs.</li></ul> |
| Possible task(s)*                                  | Functions   | From Mathematics Assessment Project  |

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# Algebra I Planning Guide – SY12-13

## Big Idea: Solving Systems of Equations and Inequalities

### Big Idea Assessment: Solving Systems of Equations and Inequalities

See Algebra I Toolset\* : Passenger Jet

### Topic: Solving Systems through Tables, Charts, and Graphs

#### CCSS-M Content Standards

- 8-EE.8a Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
- 8-EE.8b Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.

| Connections to Standards for Mathematical Practice | Standards of Mathematical Practice  | How it applies...  |
|--|---|--|
|  | MP1 - Make sense of problems and persevere in solving them.   | Analyze given information to develop possible strategies for solving the problem   |
|  | MP6 - Attend to precision.  | Formulate precise explanations (orally and in written form) using both mathematical representations and words  |
|  | MP7 - Look for and make use of structure.   | Use patterns or structure to make sense of mathematics and connect prior knowledge to similar situations and extend to novel situations  |
| Prior Knowledge                                    | Solve linear equations and inequalities in one variable.  |  |
| Students will be able to:                          | <ul style="list-style-type: none"><li>Identify the two variables needed to solve a word problem and write a system of linear equations in those two variables to model the situation.</li><li>Solve a system of two linear equations by making an appropriate table of values by hand and using technology.</li></ul> | <ul style="list-style-type: none"><li>Solve a system of two linear equations by graphing the equations and finding their point of intersection, by hand and using technology.</li><li>Check solutions to a system of two linear equations.</li></ul> |
| Possible task(s)*                                  |   |  |

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# Algebra I Planning Guide – SY12-13

## Big Idea: Solving Systems of Equations and Inequalities

### Topic: Solving Systems using Substitution and Elimination

#### CCSS-M Content Standards

- A-REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.  
A-REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

| Connections to Standards for Mathematical Practice | Standards of Mathematical Practice  | How it applies...   |
|--|---|---|
|  | MP1 - Make sense of problems and persevere in solving them.   | Analyze given information to develop possible strategies for solving the problem.   |
|  | MP6 - Attend to precision.  | Formulate precise explanations (orally and in written form) using both mathematical representations and words.                            |
|  | MP7 - Look for and make use of structure.   | Use patterns or structure to make sense of mathematics and connect prior knowledge to similar situations and extend to novel situations.  |
| Prior Knowledge                                    | <ul style="list-style-type: none"><li>Solve linear equations and inequalities in one variable.</li><li>Create a system of equations.</li></ul>  | <ul style="list-style-type: none"><li>Solve system of equations by graphing.</li><li>Solve system of equations by substitution.</li></ul> |
| Students will be able to:                          | <ul style="list-style-type: none"><li>Be able to solve systems of linear equations using the substitution method.</li><li>Be able to solve systems of linear equations using the linear combination method (elimination).</li><li>Be able to recognize dependent and inconsistent systems and write the solution set of each.</li></ul> |   |
| Possible task(s)*                                  |   |   |

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# Algebra I Planning Guide – SY12-13

## Big Idea: Solving Systems of Equations and Inequalities

### Topic: Problem Solving with Systems of Equations

#### CCSS-M Content Standards

8-EE.8c Solve real-world and mathematical problems leading to two linear equations in two variables.

| Connections to Standards for Mathematical Practice | Standards of Mathematical Practice   | How it applies...   |
|--|--|---|
|  | MP1 - Make sense of problems and persevere in solving them.  | Analyze given information to develop possible strategies for solving the problem.                                   |
|  | MP4 - Model with mathematics.  | Simplify a complicated problem by making assumptions and approximations.  |
|  | MP5 - Use appropriate tools strategically.   | Use a variety of technologies, including digital content, to explore, confirm, and deepen conceptual understanding. |
| Prior Knowledge                                    | <ul style="list-style-type: none"><li>• Solve linear equations and inequalities in one variable.</li><li>• Create a system of equations.</li></ul>   | <ul style="list-style-type: none"><li>• Solve system of equations.</li></ul>  |
| Students will be able to:                          | <ul style="list-style-type: none"><li>• Write a system of linear equations in two variables to model a problem situation.</li><li>• Determine which solution method might be most efficient for a given system of linear equations.</li><li>• Solve system of linear inequalities graphically.</li></ul> |   |
| Possible task(s)*                                  |  |   |

\*The Algebra I toolset includes this and other resources and can be found online at <https://ocs.cps.k12.il.us/sites/IKIVC/default.aspx> and on the Department of Mathematics and Science website ([cmsi.cps.k12.il.us](http://cmsi.cps.k12.il.us)).

# Algebra I Planning Guide – SY12-13

## Big Idea: Non-linear Functions and Equations

### Big Idea Assessment: Non-linear Functions and Equations See Algebra I Toolset\* : Summer Olympics

#### Topic: Exponents and Exponential Functions

##### CCSS-M Content Standards

- N-RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define  $5^{\frac{1}{3}}$  to be the cube root of 5 because we want  $(5^{\frac{1}{3}})^3 = 5(5^{\frac{1}{3}})^2$  to hold, so  $(5^{\frac{1}{3}})^3$  must equal 5.
- N-RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.
- F-IF.7e Graph exponential and logarithmic functions, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.
- F-IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.
- F-LE.1a Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
- F-LE.1c Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
- F-LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

| Connections to Standards for Mathematical Practice | Standards of Mathematical Practice  | How it applies...  |
|--|---|--|
|  | MP4 - Model with mathematics.   | Simplify a complicated problem by making assumptions and approximations.   |
|  | MP7 - Look for and make use of structure.   | Use patterns or structure to make sense of mathematics and connect prior knowledge to similar situations and extend to novel situations.   |
|  | MP8 - Look for and express regularity in repeated reasoning.  | Evaluate the reasonableness of results throughout the mathematical process while attending to detail.  |
| Prior Knowledge                                    | <ul style="list-style-type: none"><li>• Write and evaluate numerical expressions.</li><li>• Know and apply the properties of exponents.</li></ul>   | <ul style="list-style-type: none"><li>• Analyze a graph where the function is increasing or decreasing.</li></ul>  |
| Students will be able to:                          | <ul style="list-style-type: none"><li>• Develop and understand the laws of exponents.</li><li>• Simplify numerical and variable expressions involving exponent.</li><li>• Develop and understand the laws of exponents.</li><li>• Simplify numerical and variable expressions involving exponent.</li><li>• Determine if a relationship represented by a table, rule, graph, or statement can be represented by an exponential function.</li><li>• Use functions of the form <math>y = ab^x</math> to represent exponential relationships.</li><li>• Explain how changes in the parameters <math>a</math> and <math>b</math> for <math>y = ab^x</math> affect the graph of an exponential function.</li></ul> | <ul style="list-style-type: none"><li>• Analyze data and represent situations involving exponential relationships.</li><li>• Reason about the relative magnitude of quantities.</li><li>• Use scientific notation to represent quantities and solve problems.</li><li>• Determine if a table, graph, rule or statement can be represented by a linear or exponential function.</li><li>• Recognize that exponential function values are generated by common multipliers, not additive constants.</li><li>• Understand the roles that the parameters <math>a</math> and <math>b</math> in the general form the exponential function <math>y = a \cdot b^x</math> play in determining the starting points and the growth or decay of the function.</li></ul> |

#### Possible task(s)\*

\*The Algebra I toolset includes this and other resources and can be found online at <https://ocs.cps.k12.il.us/sites/IKM/C/default.aspx> and on the Department of Mathematics and Science website ([cmnsi.cps.k12.il.us](http://cmnsi.cps.k12.il.us)).

## Planning Guides

# Algebra I Planning Guide – SY12-13

## Big Idea: Non-linear Functions and Equations

### Topic: Polynomial Addition and Multiplication and the Application of Operations on Polynomial Expressions

#### CCSS-M Content Standards

- A-SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients.  
A-APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
- A-SSE.3a Factor a quadratic expression to reveal the zeros of the function it defines.

#### Connections to Standards for Mathematical Practice

MP2 - Reason abstractly and quantitatively.

MP7 - Look for and make use of structure.

MP8 - Look for and express regularity in repeated reasoning.

#### Standards of Mathematical Practice

- How it applies...**
- Recognize the relationships between numbers/quantities within the process to evaluate a problem.
- Use patterns or structure to make sense of mathematics and connect prior knowledge to similar situations and extend to novel situations.
- Evaluate the reasonableness of results throughout the mathematical process while attending to detail

#### Prior Knowledge

- Write an equation.
  - Understand and know the difference between dependent and independent variables.
  - Analyze the relationship between the dependent and independent variables using a graph.
- Students will be able to:
- Classify polynomials by type and degree.
  - Model a situation with a polynomial expression.
  - Multiply monomials, binomials, and trinomials with a variety of methods, including (but not limited to) using concrete models and directly applying the distributive property.
  - Add and subtract polynomials, simplifying with a variety of methods, including (but not limited to) using concrete models and algebraically combining like terms.
  - Define rational and irrational numbers.
  - Factor quadratic expressions.

#### Possible task(s)\*

\* The Algebra I toolset includes this and other resources and can be found online at <http://ocs.cps.k12.il.us/sites/IKYC/default.aspx> and on the Department of Mathematics and Science website ([cmst.cps.k12.il.us](http://cmst.cps.k12.il.us)).

# Algebra I Planning Guide – SY12-13

## Big Idea: Non-linear Functions and Equations

### Topic: Modeling with Quadratic Functions and Solving Quadratic Equations

#### CCSS-M Content Standards

- F-IF:7a Graph linear and quadratic functions and show intercepts, maxima, and minima.
- F-IF:9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.
- A-REI:4b Solve quadratic equations by inspection (e.g., for  $x^2 = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a \pm bi$  for real numbers  $a$  and  $b$ .
- A-REI:11 Explain why the  $x$ -coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = g(x)$  intersect are the solutions of the equation  $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where  $f(x)$  and/or  $g(x)$  are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

| Connections to Standards for Mathematical Practice | Standards of Mathematical Practice  | How it applies...   |
|--|---|---|
|  | MP1 - Make sense of problems and persevere in solving them.   | Analyze given information to develop possible strategies for solving the problem.   |
|  | MP4 - Model with mathematics.   | Use a variety of methods to model, represent, and solve real-world problems.  |
|  | MP5 - Use appropriate tools strategically.  | Select and use appropriate tools to best model/solve problems.  |
| Prior Knowledge                                    | <ul style="list-style-type: none"><li>• Identify rational and irrational numbers</li><li>• Approximate irrational numbers</li></ul>   | <ul style="list-style-type: none"><li>• Solve equations and inequalities</li><li>• Create equations and inequalities</li></ul>  |
| Students will be able to:                          | <ul style="list-style-type: none"><li>• Determine if a relationship represented by a table, rule, graph, or statement can be represented by a quadratic function.</li><li>• Use functions of the form <math>y = ax^2 + c</math> to represent some quadratic relationships.</li><li>• Explain how changes in the parameters <math>a</math> and <math>c</math> for <math>y = ax^2 + c</math> affect the graph of the parent quadratic function <math>y = x^2</math>.</li><li>• Identify and make connections between solutions and <math>x</math>-intercepts.</li><li>• Simplify square roots algebraically and connect the simplified form to the geometric models for square roots.</li><li>• Use the discriminant to determine the number of solutions for a quadratic equation.</li></ul> | <ul style="list-style-type: none"><li>• Solve quadratic equations by factoring</li><li>• Identify and make connections among factors, solutions, <math>x</math>-intercepts, and zeros.</li><li>• Solve quadratics by graphing.</li><li>• Explain the meaning of solutions for given situations</li><li>• Solve quadratic equations using the quadratic formula.</li><li>• Use the discriminant to determine the number of solutions for a quadratic equation.</li><li>• Explain the meaning of solutions for given situation.</li></ul> |
| Possible task(s)*                                  |   |   |

\* The Algebra I toolset includes this and other resources and can be found online at <https://ocs.cps.k12.il.us/sites/IKIVC/default.aspx> and on the Department of Mathematics and Science website ([cmst.cps.k12.il.us](http://cmst.cps.k12.il.us)).

# Geometry Planning Guide – SY12-13

## Introduction

The purpose of this Geometry Planning Guide is to define the scope of the Common Core State Standards for Mathematics (CCSS-M) Content Standards to be taught in the 2012-2013 school year. It has been designed by CPS teachers to be useful to CPS teachers during the three-year transition to full implementation of the CCSS-M.

In general, a Geometry course formalizes and extends students' geometric experiences from the middle grades. In Year 1 of our transition to CCSS-M, 2012-2013, the Geometry Planning Guide focuses student learning on defining geometric shapes, points, lines, and planes, concepts found in the CCSS-M for Grades 7 and 8. Because this focus on geometric construction is new for many CPS students, we have allocated more time than is provided for in the traditional Geometry course outlined in CCSS-M Appendix A: *Designing High School Mathematics Courses on the Common Core State Standards* (2011). To accommodate this, the Applications of Probability unit (found in the Appendix A course outline) will not be included in Year 1, but will be included in Year 2 (for 2013-2014).

This Planning Guide includes five Big Ideas (below). For each Big Idea, a summative assessment is referenced, and topics provide the detailed components to support instruction. Topic components include a targeted set of content standards, target mathematical practices (and how they apply to the specific topic), and sample instructional tasks. The sample tasks have been selected for their rigor and their potential usefulness as formative performance assessments for the topic. A toolset to support instruction includes: samples of lesson planning templates; samples of rigorous tasks; samples of formative assessment options based on MARS (Mathematics Assessment Resource Service) tasks; tools for examining and modifying lessons/tasks to increase rigor; samples of modified lessons/tasks; and a professional resource list. These tools are available at <https://ocs.cps.k12.il.us/sites/IKMC/default.aspx> and on the Department of Mathematics and Science website ([cmsi.cps.k12.il.us](http://cmsi.cps.k12.il.us)).

The guide assumes 156 days for instruction, including time for formative and summative assessments. The Big Ideas are sequenced in a way that we believe best develops and connects the mathematical content of the CCSS-M. However, teachers should review the Big Ideas and decide the order and time allocation appropriate for their classrooms, given their students, instructional materials, and other considerations. The order of the standards presented in a topic **does not imply a sequence of the content**. Some standards may be revisited several times while addressing the topic, while others may be only partially addressed, depending on the topic's mathematical focus.

Throughout the Geometry course, students should continue to develop proficiency in the Common Core's Standards for Mathematical Practice (below). Teachers should integrate the instruction of content standards with mathematical practices. This Planning Guide includes "how to" guidance to support this approach to integrated instruction. When the mathematical practices are taught alongside the indicated content standards, students will be more likely to achieve the depth of conceptual understanding and procedural fluency that are expected by the CCSS-M.

## The CCSS-M Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning

Finally, this document reflects our current thinking about the intent of CCSS-M, but we recognize that we will learn more as we begin the transition. This learning will be reflected in refinements to the materials. Please share your implementation insights with us, so that the District can benefit from your experience. Network Instructional Support Leaders (ISLs) provide the interface between you, the teacher, and the Department of Mathematics and Science.

| BIG IDEAS in GEOMETRY, YEAR 1                            | PAGE |
|--|------|
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| Extending to Three Dimensions .....                      | 77   |
| Connecting Algebra and Geometry through Coordinates..... | 80   |
| Circles With and Without Coordinates.....                | 82   |

# Geometry Planning Guide – SY12-13

## Big Idea: Congruence, Proof, and Constructions

### Big Idea Assessment: Congruence, Proof, and Constructions See Geometry Toolset\*: Using Tools of Geometry

#### Topic: Introduction to Geometry

##### CCSS-M Content Standards

G-CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

| Connections to Standards for Mathematical Practice |   | Standards of Mathematical Practice   | How it applies...   |
|--|---|--|---|
|  |   | MP5 - Use appropriate tools strategically.   | Identify mathematical tools and recognize their strengths and weaknesses  |
|  |   | MP6 - Attend to precision.   | Communicate using clear mathematical definitions, vocabulary, and symbols |
| Prior Knowledge                                    |   | <ul style="list-style-type: none"><li>Definitions of: Acute Angles, Obtuse Angles, Right Angles, Acute Triangle, Obtuse Triangle, Right Triangle, Isosceles Triangles, Scalene Triangles, Equilateral Triangle, Polygons, Regular Polygons, Trapezoids, Isosceles Trapezoids</li></ul> |   |
| Students will be able to:                          | <ul style="list-style-type: none"><li>Name and/or use the correct symbolic notational conventions for: Angle, Measure of an Angle, Circle, Perpendicular, Parallel, Segment, Measure of a Segment, Point, Line, Plane, Endpoint, Ray, Plane, Angle Bisector, Midpoint, Segment Bisectors, Perpendicular Bisectors, Betweenness, Congruent Segments, Congruent Angles, Tick Marks, Length of Segment, Measure of an Angle, Collinear, Angle Addition, Segment Addition</li></ul> |  |   |
| Possible tasks*                                    |   |  |   |

\* The Geometry toolset includes this and other resources and can be found online at <https://ocs.cps.k12.il.us/sites/KMC/default.aspx> and on the Department of Mathematics and Science website ([cmsi.cps.k12.il.us](http://cmsi.cps.k12.il.us)).

# Geometry Planning Guide – SY12-13

## Big Idea: Congruence, Proof, and Constructions

### Topic: Tools of Geometry

#### CCSS-M Content Standards

- G-CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.
- G-CO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

| Connections to Standards for Mathematical Practice | Standards of Mathematical Practice   | How it applies...   |
|--|--|---|
|  | MP1 - Make sense of problems and persevere in solving them.  | Check for accuracy and reasonableness of work, strategy and solution      |
|  | MP5 - Use appropriate tools strategically.   | Select and use appropriate tools to best model/solve problems             |
|  | MP6 - Attend to precision.   | Calculate answers efficiently and accurately and label them appropriately |
| Prior Knowledge                                    | <ul style="list-style-type: none"><li>Definitions of Congruent Segments, Congruent Angles, Radius, Arc, Midpoint, Bisector, Perpendicular Lines, Parallel Lines, Perpendicular Bisector, Equilateral Triangle, Square, Perpendicular, Equidistant</li></ul>  |   |
| Students will be able to:                          | <ul style="list-style-type: none"><li>Accurately construct: a) a segment congruent to a given segment; b) an angle to congruent to a given angle; c) an angle bisector of a given angle;</li><li>d) the segment bisector and perpendicular bisector of a given segment; e) a line perpendicular to a given line through a point on the given line; f) a line perpendicular to a given line through a point not on the given line; g) a line parallel to a given line through a given point</li><li>Accurately construct: a) an equilateral triangle given a side length; b) a square given a side length</li></ul> |   |
| Possible task(s)*                                  | See CME Project Geometry: Page 7 Model the Problem   |   |

\*The Geometry toolset includes this and other resources and can be found online at <https://ocss.cps.k12.il.us/sites/KM/C/default.aspx> and on the Department of Mathematics and Science website ([cmsi.cps.k12.il.us](http://cmsi.cps.k12.il.us)).

# Geometry Planning Guide – SY12-13

## Big Idea: Congruence, Proof, and Constructions

### Topic: Transformations

#### CCSS-M Content Standards

- G-CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
- G-CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
- G-CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
- G-CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

| Connections to Standards for Mathematical Practice | Standards of Mathematical Practice  | How it applies...  |
|--|---|--|
|  | MP4 - Model with mathematics.   | Choose a model that is both appropriate and efficient to arrive at one or more desired solutions.  |
|  | MP5 - Use appropriate tools strategically.  | Identify and successfully use external mathematical resources to pose or solve problems.   |
|  | MP6 - Attend to precision.  | Calculate answers efficiently and accurately and label them appropriately  |
|  | MP7 - Look for and make use of structure.   | Look for, identify, and accept patterns or structure within relationships  |
|  | MP8 - Look for and express regularity in repeated reasoning.  | Generalize the process to create a shortcut which may lead to developing rules or creating a formula.  |
| Prior Knowledge                                    | <ul style="list-style-type: none"> <li>• Segment length and angle measure</li> <li>• Definitions of parallelogram, rectangle, trapezoid, and regular polygon</li> </ul>   | <ul style="list-style-type: none"> <li>• Students should be able to accurately construct perpendicular lines through a point not on the line, congruent angles, and congruent segments</li> </ul>  |
| Students will be able to:                          | <ul style="list-style-type: none"> <li>• Describe the different types of transformations and be able to differentiate between them.</li> <li>• Understand the difference between transformations that are isometries and those that are not.</li> <li>• Identify and construct the line(s) of symmetry, if they exist, for any polygon.</li> <li>• Identify and locate the center of rotation for any polygon, if it exists, and determine the degree of rotational symmetry.</li> <li>• Classify objects as n-fold reflectional and/or n-rotational and be able to determine the value of n for each.</li> <li>• Reflect an object over a given line by constructing perpendicular lines and congruent segments or by using technology.</li> </ul> | <ul style="list-style-type: none"> <li>• Rotate an object by reflecting it over two intersecting lines with and without technology.</li> <li>• Translate an object by reflecting it over two parallel lines with and without technology.</li> <li>• Rotate an object about a given center with a given angle and direction with and without technology.</li> <li>• Translate an object a given distance and given direction on the coordinate plane and geometric plane with and without technology.</li> <li>• Describe and perform the composition of transformations that will map a given object onto a congruent object in the plane with or without technology.</li> </ul> |

Possible task(s)\*

CME Project Geometry : Pages 537 and 538 in Class Experiment and For You to Explore

\*The Geometry toolset includes this and other resources and can be found online at <https://ots.cps.k12.il.us/sites/kM/C/default.aspx> and on the Department of Mathematics and Science website ([cms.cps.k12.il.us](http://cms.cps.k12.il.us)).

# Geometry Planning Guide – SY12-13

## Big Idea: Congruence, Proof, and Constructions

### Topic: Proof

#### CCSS-M Content Standards

- G-CO.9 Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
- G-CO.1 Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to  $180^\circ$ ; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
- G-CO.11 Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

| Connections to Standards for Mathematical Practice | Standards of Mathematical Practice   | How it applies...   |
|--|--|---|
|  | MP1 - Make sense of problems and persevere in solving them.  | Analyze given information to develop possible strategies for solving the problem  |
|  | MP2 - Reason abstractly and quantitatively.  | Translate given information to create a mathematical representation for a concept.  |
|  | MP3 - Construct viable arguments and critique the reasoning of others.   | Justify (orally and in written form) the approach used, including how it fits in the context from which the data arose  |
| Prior Knowledge                                    | <ul style="list-style-type: none"><li>Understand and apply the definitions of: Linear Pair of Angles, Parallel Lines and Angles Formed by Transversals, Corresponding Angles, Corresponding Sides, Perpendicular Lines, Perpendicular Bisector, and Equidistance</li><li>Understand and apply the definitions of: Parallelograms, Diagonals, Opposite Sides, Opposite Angles, Consecutive Angles, Rectangles, Bisect</li></ul> | <ul style="list-style-type: none"><li>Understand and apply the definitions of: Interior Angles, Isosceles Triangles, Parts of Isosceles Triangles, Midsegment of a Triangle, Medians of a Triangle</li><li>Mid-segment of a triangle parallel to a side and half the length</li><li>Medians of a triangle are concurrent</li><li>Opposite sides of a parallelogram are congruent</li><li>Opposite angles of a parallelogram are congruent</li><li>Diagonals of a parallelogram bisect each other</li><li>Rectangles are parallelograms with congruent diagonals</li></ul> |
| Students will be able to:                          | <ul style="list-style-type: none"><li>Prove the following theorems:<ul style="list-style-type: none"><li>Vertical angle theorem</li><li>Alternate interior angle theorem</li><li>Corresponding angle theorem</li><li>Perpendicular bisector theorem</li><li>Triangle sum theorem</li><li>Isosceles triangle theorem</li></ul></li></ul>  | <ul style="list-style-type: none"><li>Vertical angle theorem</li><li>Alternate interior angle theorem</li><li>Corresponding angle theorem</li><li>Perpendicular bisector theorem</li><li>Triangle sum theorem</li><li>Isosceles triangle theorem</li></ul>  |
| Possible task(s)*                                  | See Carnegie Learning, Inc. Geometry: Pages 91 and 92 What's Your Proof?   |   |

\* The Geometry toolset includes this and other resources and can be found online at <https://ocs.cps.k12.il.us/sites/KIMC/default.aspx> and on the Department of Mathematics and Science website ([cmsi.cps.k12.il.us](http://cmsi.cps.k12.il.us)).

# Geometry Planning Guide – SY12-13

## Big Idea: Similarity & Congruence

### Big Idea Assessment: Similarity & Congruence See Geometry Toolset\*: Montrose High Rocket Club

#### Topic: Similarity

##### CCSS-M Content Standards

- G-SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
- G-SRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.
- G-SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

| Connections to Standards for Mathematical Practice | Standards of Mathematical Practice   | How it applies...   |
|--|--|---|
|  | MP3 - Construct viable arguments and critique the reasoning of others.   | Use observations and prior knowledge (stated assumptions, definitions, and previous established results) to make conjectures and construct arguments  |
|  | MP4 - Model with mathematics.  | Simplify a complicated problem by making assumptions and approximations   |
|  | MP5 - Use appropriate tools strategically.   | Select and use appropriate tools to best model/solve problems   |
|  | MP7 - Look for and make use of structure.  | Look for, identify, and accept patterns or structure within relationships   |
| Prior Knowledge                                    | <ul style="list-style-type: none"><li>Proportional reasoning and using transformations</li><li>Basic ability to mathematically support a prediction or hypothesis</li></ul>  | <ul style="list-style-type: none"><li>Recognize a situation's connection to mathematics model</li></ul>   |
| Students will be able to:                          | <ul style="list-style-type: none"><li>Solve linear equations in one variable.</li><li>Identify linear equations with one, infinitely many, or no solution.</li><li>Transform an expression or equation into alternate forms.</li></ul> | <ul style="list-style-type: none"><li>Find the solution to a system of equations from a graph.</li><li>Solve systems algebraically using substitution and elimination/combination.</li><li>Understand the solution of a system of equations in the context of the problem and write as an ordered pair.</li></ul> |
| Possible task(s)*                                  |  |   |

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# Geometry Planning Guide – SY12-13

## Big Idea: Similarity & Congruence

### Topic: Similarity in Dilations

#### CCSS-M Content Standards

- G-SRT.1 Verify experimentally the properties of dilations given by a center and a scale factor:  
a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.  
b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor

| Connections to Standards for Mathematical Practice |  | Standards of Mathematical Practice   | How it applies...   |
|--|--|--|---|
|  |  | MP1 -Make sense of problems and persevere in solving them.   | Identify and execute appropriate strategies to solve the problem  |
|  |  | MP3 - Construct viable arguments and critique the reasoning of others.   | Justify (orally and in written form) the approach used, including how it fits in the context from which the data arose  |
|  |  | MP8 - Look for and express regularity in repeated reasoning.   | Generalize the process to create a shortcut which may lead to developing rules or creating a formula  |
| Prior Knowledge                                    |  | <ul style="list-style-type: none"><li>• Fluency with similarity of two-dimensional figures. (implies fluency with ratios and proportional reasoning)</li><li>• Parallel lines cut by traversals properties or establish the properties</li></ul>   |   |
| Students will be able to:                          |  | <ul style="list-style-type: none"><li>• Given a one- or two-dimensional figure in a plane and a point on or not on the plane, create specific dilations.</li><li>• Be able to explain (justify) the relation between areas of a figure and its dilation (for simple figures: triangles, quadrilaterals, and hexagons or other simple figures).</li></ul> | <ul style="list-style-type: none"><li>• Some students might enjoy extending dilations to three-dimensional solids and explaining the relation between the original solid and the dilated solid.</li></ul> |
|  |  | Possible task(s)*  |   |

\* The Geometry toolset includes this and other resources and can be found online at <https://ocs.cps.k12.il.us/sites/KMC/default.aspx> and on the Department of Mathematics and Science website ([cmsi.cps.k12.il.us](http://cmsi.cps.k12.il.us)).

# Geometry Planning Guide – SY12-13

## Big Idea: Similarity & Congruence

### Topic: Similarity and Trigonometric Ratios

#### CCSS-M Content Standards

G-SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

★

G-SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

★

G-SRT.7 Explain and use the relationship between the sine and cosine of complementary angles.

★

Specific modeling standard (versus an example of the modeling Standard for Mathematical Practice).

| Connections to Standards for Mathematical Practice | Standards of Mathematical Practice  |   | How it applies...  |
|--|---|---|--|
|  | MP1 - Make sense of problems and persevere in solving them.   | MP2 - Reason abstractly and quantitatively.   |  |
|  | MP3 - Construct viable arguments and critique the reasoning of others.  | Analyze given information to develop possible strategies for solving the problem  | Justify (orally and in written form) the approach used, including how it fits in the context from which the data arose |
|  | MP4 - Model with mathematics.   | Use a variety of methods to model, represent, and solve real-world problems   |  |
|  | MP8 - Look for and express regularity in repeated reasoning.  | Recognize similarities and patterns in repeated trials with a process   |  |
| Prior Knowledge                                    | <ul style="list-style-type: none"> <li>Fluency with ratios and proportional reasoning</li> <li>Fluency with dilations</li> </ul>  | <ul style="list-style-type: none"> <li>Recognize a situation's connection to mathematics model</li> <li>Basic ability to mathematically support a prediction or hypothesis</li> </ul>   |  |
| Students will be able to:                          | <ul style="list-style-type: none"> <li>Explain/justify that the sine, cosine, and tangent of any given acute angle in a right triangle are constant for any pair of dilated triangles.</li> <li>Identify the opposite side of an angle, the adjacent side of an angle, and the hypotenuse in any right triangle.</li> <li>Determine the specific ratios for sine, cosine, and tangent for a specified angle in a right triangle if all the sides are known or if only two sides are known.</li> </ul> | <ul style="list-style-type: none"> <li>Apply trigonometry to triangles or real world situations.</li> <li>Understand and be able to explain when the sine and cosine have the same value for different angles and explain the relationship between these angles.</li> </ul> |  |
| Possible task(s)*                                  | Hopewell Triangles  | From: Mathematics Assessment Project  |  |

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# Geometry Planning Guide – SY12-13

## Big Idea: Extending to Three Dimensions

**Big Idea Assessment: Extending to Three Dimensions**  
See Geometry Toolset\*: Dream House

### Topic: Volume Formulas and Problem Solving

#### CCSS-M Content Standards

G-GMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.

G-GMD.2 (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

G-GMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★

+ Additional content that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics  
★ Specific modeling standard (versus an example of the modeling Standard for Mathematical Practice)

| Connections to Standards for Mathematical Practice | Standards of Mathematical Practice  | How it applies...  |
|--|---|--|
|  | MP3 - Construct viable arguments and critique the reasoning of others.  | Use observations and prior knowledge (stated assumptions, definitions, and previous established results) to make conjectures and construct arguments   |
|  | MP4 - Model with mathematics.   | Simplify a complicated problem by making assumptions and approximations  |
|  | MP7 - Look for and make use of structure.   | Use patterns or structure to make sense of mathematics and connect prior knowledge to similar situations and extend to novel situations  |
|  | MP8 - Look for and express regularity in repeated reasoning.  | Evaluate the reasonableness of results throughout the mathematical process while attending to detail   |
| Prior Knowledge                                    | <ul style="list-style-type: none"><li>This unit builds on the 7th grade geometry standard: 7-G Solve real-life and mathematical problems involving angle measure, area, surface area, and volume</li><li>Algebra manipulations and reasoning skills</li></ul> | <ul style="list-style-type: none"><li>Basic understanding of area as the square units of covering a given figure or shape and be able to calculate areas and perimeters of simple figures; rectangles, triangles, and possibly circles</li></ul> |
| Students will be able to:                          | <ul style="list-style-type: none"><li>Derive the formulas for circumference, perimeter, area, and volume for various geometric figures with and without the use of technology.</li><li>Apply the derived formulas.</li></ul>                                  |  |
| Possible task(s)*                                  | Glasses   | From: Mathematics Assessment Project   |

\*The Geometry toolset includes this and other resources and can be found online at <https://ocss.cps.k12.il.us/sites/kIMC/default.aspx> and on the Department of Mathematics and Science website ([cmsi.cps.k12.il.us](http://cmsi.cps.k12.il.us)).

# Geometry Planning Guide – SY12-13

## Big Idea: Extending to Three Dimensions

### Topic: Visualize Similarities Between Two- and Three-Dimensional Objects

#### CCSS-M Content Standards

G-GMD.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

| Connections to Standards for Mathematical Practice | Standards of Mathematical Practice  | How it applies...  |
|--|---|--|
|  | MP1 - Make sense of problems and persevere in solving them.   | Analyze given information to develop possible strategies for solving the problem   |
|  | MP3 - Construct viable arguments and critique the reasoning of others.  | Use observations and prior knowledge (stated assumptions, definitions, and previous established results) to make conjectures and construct arguments |
| Prior Knowledge                                    | <ul style="list-style-type: none"><li>Know the difference between two- and three-dimensional objects</li><li>Visualization skills and/or tools to help visualize. For example, clay models to be cut by a string to “see” what the cross sectional area looks like</li></ul>  |  |
| Students will be able to:                          | <ul style="list-style-type: none"><li>Given a specific three-dimensional shape, identify and explain planar cross sections of that shape.</li><li>Describe the three-dimensional object generated when any two-dimensional figure is rotated about:<ol style="list-style-type: none"><li>either axis</li><li>any line parallel to either axis</li></ol></li></ul> |  |
| Possible task(s)*                                  | Propane Tank  | From: Mathematics Assessment Project   |

\*The Geometry toolset includes this and other resources and can be found online at <https://oer.cps.k12.il.us/sites/KMC/default.aspx> and on the Department of Mathematics and Science website ([cms.k12.il.us](http://cms.k12.il.us)).

# Geometry Planning Guide – SY12-13

## Big Idea: Extending to Three Dimensions

### Topic: Applying Geometric Concepts in Modeling Situations

#### CCSS-M Content Standards

G-MG.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). ★

★Specific modeling standard (versus an example of the modeling Standard for Mathematical Practice)

| Connections to Standards for Mathematical Practice | Standards of Mathematical Practice   | How it applies...   |
|--|--|---|
|  | MP1 - Make sense of problems and persevere in solving them.  | Identify and execute appropriate strategies to solve the problem  |
|  | MP4 - Model with mathematics.  | Choose a model that is both appropriate and efficient to arrive at one or more desired solutions  |
|  | MP6 - Attend to precision.   | Communicate using clear mathematical definitions, vocabulary, and symbols   |
|  | MP7 - Look for and making use of structure.  | Use patterns or structure to make sense of mathematics and connect prior knowledge to similar situations and extend to novel situations |
| Prior Knowledge                                    | <ul style="list-style-type: none"><li>• Know the difference between two- and three-dimensional objects</li><li>• Visualization skills and/or tools to help visualize. For example, clay models to be cut by a string to “see” what the cross sectional area looks like</li></ul> |   |
| Students will be able to:                          | <ul style="list-style-type: none"><li>• Draw any three-dimensional object using only geometric shapes.</li></ul>   |   |
| Possible task(s)*                                  |  |   |

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# Geometry Planning Guide – SY12-13

## Big Idea: Connecting Algebra and Geometry through Coordinates

**Big Idea Assessment: Connecting Algebra and Geometry Through Coordinates**  
See Geometry Toolset\*: Neighborhood Map

### Topic: Slope Formula

#### CCSS-M Content Standard

G-GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

| Connections to Standards for Mathematical Practice | Standards of Mathematical Practice  | How it applies...  |
|--|---|--|
|  | MP3 - Construct viable arguments and critique the reasoning of others.  | Use observations and prior knowledge (stated assumptions, definitions, and previous established results) to make conjectures and construct arguments       |
|  | MP5 - Use appropriate tools strategically.  | Select and use appropriate tools to best model/solve problems  |
|  | MP6 - Attend to precision.  | Calculate answers efficiently and accurately and label them appropriately  |
| Prior Knowledge                                    | <ul style="list-style-type: none"><li>• Slope (conceptually, graphically, algebraically)</li><li>• Rotations and translations of coordinate points</li></ul>        | <ul style="list-style-type: none"><li>• Slope-intercept form</li></ul>   |
| Students will be able to:                          | <ul style="list-style-type: none"><li>• Find the slope between two given points.</li><li>• Prove the slope criteria for parallel and perpendicular lines.</li></ul> | <ul style="list-style-type: none"><li>• Find the equation of a line parallel or perpendicular to a given line that passes through a given point.</li></ul> |
| Possible task(s)*                                  |   |  |

\* The Geometry toolset includes this and other resources and can be found online at <https://ocs.cps.k12.il.us/sites/KMC/default.aspx> and on the Department of Mathematics and Science website ([cmsi.cps.k12.il.us](http://cmsi.cps.k12.il.us)).

# Geometry Planning Guide – SY12-13

## Big Idea: Connecting Algebra and Geometry through Coordinates

| Topic: Distance Formula   | CCSS-M Content Standards   | Connections to Standards for Mathematical Practice  | Standards of Mathematical Practice   | How it applies.....  |
|---------------------------|--|---|--|--|
| G-GPE.6                   | Find the point on a directed line segment between two given points that partition the segment in a given ratio.  |   |  |  |
| G-GPE.7                   | Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.   |   |  |  |
| G-GPE.4                   | Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle.                      |   |  |  |
| Prior Knowledge           | <ul style="list-style-type: none"><li>• Distance on a number line</li><li>• Pythagorean Theorem</li><li>• Ratio</li></ul>  | <ul style="list-style-type: none"><li>• MP3 - Construct viable arguments and critique the reasoning of others.</li><li>• MP4 - Model with mathematics.</li><li>• MP6 - Attend to precision.</li></ul> | <ul style="list-style-type: none"><li>• MP1 - Make sense of problems and persevere in solving them.</li><li>• MP2 - Reason abstractly and quantitatively.</li><li>• MP5 - Use appropriate tools strategically.</li></ul> | <ul style="list-style-type: none"><li>• Use observations and prior knowledge (stated assumptions, definitions, and previous established results) to make conjectures and construct arguments</li><li>• Use a variety of methods to model, represent, and solve real-world problems</li><li>• Calculate answers efficiently and accurately and label them appropriately</li></ul> |
| Students will be able to: | <ul style="list-style-type: none"><li>• Find the distance between two points in the coordinate plane.</li><li>• Find the point on a line segment that partitions the segment in a given ratio.</li></ul> |   |  | <ul style="list-style-type: none"><li>• Use the distance formula to compute perimeters of polygons and areas of triangles and rectangles.</li><li>• Use the distance and slope formulas to prove that a figure is a rectangle, parallelogram, rhombus, trapezoid, kite, or square.</li></ul>   |
| Possible task(s)          |  |   |  |  |

\* The Geometry toolset includes this and other resources and can be found online at <https://ocs.cps.k12.il.us/sites/KIMC/default.aspx> and on the Department of Mathematics and Science website ([cmsi.cps.k12.il.us](http://cmsi.cps.k12.il.us)).

# Geometry Planning Guide – SY12-13

## Big Idea: Circles With and Without Coordinates

**Big Idea Assessment: Circles With and Without Coordinates**  
See Geometry Toolset\*: Circular Reasoning

### Topic: Defining a Circle and Parts of Circles

#### CCSS-M Content Standards

G-C.1 Prove that all circles are similar.

| Connections to Standards for Mathematical Practice | Standards of Mathematical Practice   | How it applies...  |
|--|--|--|
|  | MP3 - Construct viable arguments and critique the reasoning of others.   | Use observations and prior knowledge (stated assumptions, definitions, and previous established results) to make conjectures and construct arguments |
|  | MP7 - Look for and make use of structure.  | Use patterns or structure to make sense of mathematics and connect prior knowledge to similar situations and extend to novel situations              |
| Prior Knowledge                                    | <ul style="list-style-type: none"><li>Definitions of: Circle, Radius, Diameter, Circumference, Similarity</li></ul>  |  |
| Students will be able to:                          | <ul style="list-style-type: none"><li>Identify (and name) radii, diameters, and chords.</li><li>Prove that all circles are similar (<i>by comparing ratios of radii and circumference of different circles and proving that a constant scale factor exists</i>).</li></ul> |  |
| Possible task(s)                                   |  |  |

\*The Geometry toolset includes this and other resources and can be found online at <https://ocs.cps.k12.il.us/sites/KMC/default.aspx> and on the Department of Mathematics and Science website ([cmsi.cps.k12.il.us](http://cmsi.cps.k12.il.us)).

# Geometry Planning Guide – SY12-13

## Big Idea: Circles With and Without Coordinates

### Topic: Relationships Among Inscribed Angles, Radii, and Chords

#### CCSS-M Content Standards

G-C.2 Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent line where the radius intersects the circle.*

| Connections to Standards for Mathematical Practice | Standards of Mathematical Practice  | How it applies...  |
|--|---|--|
|  | MP3 - Construct viable arguments and critique the reasoning of others.  | Use observations and prior knowledge (stated assumptions, definitions, and previous established results) to make conjectures and construct arguments   |
|  | MP7 - Look for and make use of structure.   | Look for, identify, and accept patterns or structure within relationships  |
|  | MP8 - Look for and express regularity in repeated reasoning.  | Generalize the process to create a shortcut which may lead to developing rules or creating a formula   |
| Prior Knowledge                                    |   |  |
| Students will be able to:                          | <ul style="list-style-type: none"><li>• Identify tangent lines, inscribed angles, and central angles.</li><li>• Apply the relationship between two congruent inscribed angles and their intercepted arcs.</li><li>• Identify (and apply) that an inscribed angle is one half of the measure of the intercepted arc or central angle that intersects the same arc.</li></ul> | <ul style="list-style-type: none"><li>• Identify (and apply) that a circumscribed angle is the supplement of the central angle with the same intercepted arc.</li><li>• Prove that an inscribed angle in a semi-circle is a right angle.</li><li>• Delineate (and apply) how the radius of a circle is perpendicular to the tangent line at the point of tangency.</li></ul> |
| Possible task(s)*                                  |   |  |

\* The Geometry toolset includes this and other resources and can be found online at <https://ocs.cps.k12.il.us/sites/KM/C/default.aspx> and on the Department of Mathematics and Science website ([cmsi.cps.k12.il.us](http://cmsi.cps.k12.il.us)).

# Geometry Planning Guide – SY12-13

## Big Idea: Circles With and Without Coordinates

### Topic: Constructions

#### CCSS-M Content Standards

G-C.3 Construct the inscribed and circumscribed circles of a triangle and prove properties of angles for a quadrilateral inscribed in a circle.

| Connections to Standards for Mathematical Practice | Standards of Mathematical Practice  | How it applies...  |
|--|---|--|
|  | MP3 - Construct viable arguments and critique the reasoning of others.  | Use observations and prior knowledge (stated assumptions, definitions, and previous established results) to make conjectures and construct arguments |
|  | MP5 - Use appropriate tools strategically.  | Select and use appropriate tools to best model/solve problems  |
|  | MP6 - Attend to precision.  | Formulate precise explanations (orally and in written form) using both mathematical representations and words  |
| Prior Knowledge                                    | <ul style="list-style-type: none"><li>• Construct a perpendicular bisector</li><li>• Bisect an angle</li></ul>  | <ul style="list-style-type: none"><li>• Construct a perpendicular</li></ul>  |
| Students will be able to:                          | <ul style="list-style-type: none"><li>• Construct the inscribed and circumscribed circles of a triangle.</li><li>• Prove that the opposite angles of a quadrilateral inscribed in a circle are supplementary.</li></ul> |  |
| Possible task(s)*                                  |   |  |

\* The Geometry toolset includes this and other resources and can be found online at <https://ocs.cps.k12.il.us/sites/KM/C/default.aspx> and on the Department of Mathematics and Science website ([cmsi.cps.k12.il.us](http://cmsi.cps.k12.il.us)).

# Geometry Planning Guide – SY12-13

## Big Idea: Circles With and Without Coordinates

| Topic: Arc Lengths and Areas of Sectors            |  |
|--|--|
| CCSS-M Content Standards                           |  |
| G-C.5  | Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.  |
| Connections to Standards for Mathematical Practice | Standards of Mathematical Practice   |
|  | MP3 - Construct viable arguments and critique the reasoning of others.<br>MP7 - Look for and make use of structure.<br>MP8 - Look for and express regularity in repeated reasoning.  |
| Prior Knowledge                                    | <ul style="list-style-type: none"><li>• Circumference and area of a circle</li><li>• Set up proportions</li></ul>  |
| Students will be able to:                          | <ul style="list-style-type: none"><li>• Find the circumference and area of a circle.</li><li>• Derive, using similarity, the fact that the length of the arc intercepted by an angle is proportional to the radius.</li><li>• Define the radian measure of the angle as the constant of proportionality.</li></ul> |
| Possible task(s)*                                  |  |

\*The Geometry toolset includes this and other resources and can be found online at <https://ocs.cps.k12.il.us/sites/KM/C/default.aspx> and on the Department of Mathematics and Science website ([cms1.cps.k12.il.us](http://cms1.cps.k12.il.us)).

# Geometry Planning Guide – SY12-13

## Big Idea: Circles With and Without Coordinates

### Topic: Coordinate Geometry of Circles

#### CCSS-M Content Standards

- G-GPE.1 Derive the equation of a circle given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.  
G-GPE.4 Use coordinates to prove simple geometric theorems algebraically. (*For example, prove or disprove that the point (1, ) lies on the circle centered at the origin and containing the point (0, 2).*)

| Connections to Standards for Mathematical Practice | Standards of Mathematical Practice   | How it applies...  |
|--|--|--|
|  | MP3 - Construct viable arguments and critique the reasoning of others.   | Use observations and prior knowledge (stated assumptions, definitions, and previous established results) to make conjectures and construct arguments |
|  | MP6 - Attend to precision.   | Formulate precise explanations (orally and in written form) using both mathematical representations and words  |
| Prior Knowledge                                    | <ul style="list-style-type: none"><li>• Pythagorean Theorem (and/or distance formula)</li><li>• Completing the square</li></ul>  |  |
| Students will be able to:                          | <ul style="list-style-type: none"><li>• Derive the equation of a circle given the center and radius using the Pythagorean Theorem.</li><li>• Complete the square to find the center and radius of a circle given by an equation.</li></ul> |  |
| Possible task(s)*                                  |  |  |

\*The Geometry toolset includes this and other resources and can be found online at <https://ocs.cps.k12.il.us/sites/KMC/default.aspx> and on the Department of Mathematics and Science website ([cmsi.cps.k12.il.us](http://cmsi.cps.k12.il.us)).

## Sample Tasks

The following sample tasks provide examples of rigorous tasks that support the expectations of the CCSS-M.

Complete Toolsets for grades 6-8 as well as Algebra I and Geometry will be available on our Knowledge Management site: <https://ocs.cps.k12.il.us/sites/IKMC> and on the Department of Mathematics and Science website: <http://cmsi.cps.k12.il.us/>.

## Grade 8 Sample Task

### Sorting Functions

This problem gives you the chance to:

- Find relationships between graphs, equations, tables and rules
- Explain your reasons

On the next page are four graphs, four equations, four tables, and four rules. Your task is to match each graph with an equation, a table and a rule.

1. Write your answers in the following table.

| Graph | Equation | Table | Rule |
|-------|----------|-------|------|
| A     |          |       |      |
| B     |          |       |      |
| C     |          |       |      |
| D     |          |       |      |

2. Explain how you matched each of the four graphs to its equation.

*Graph A* \_\_\_\_\_  
\_\_\_\_\_

*Graph B* \_\_\_\_\_  
\_\_\_\_\_

*Graph C* \_\_\_\_\_  
\_\_\_\_\_

*Graph D* \_\_\_\_\_  
\_\_\_\_\_

## Grade 8 Sample Task

|                    |                                  |   |     |    |    |     |   |    |     |     |    |    |    |    |   |     |   |
|--------------------|----------------------------------|---|-----|----|----|-----|---|----|-----|-----|----|----|----|----|---|-----|---|
| <b>Graph A</b><br> | <b>Equation A</b><br>$xy = 2$    | <b>Table A</b><br><table border="1"> <tbody> <tr> <td><math>x</math></td> <td>-2</td> <td>-1</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> </tr> <tr> <td><math>y</math></td> <td>-4</td> <td>-3</td> <td>-2</td> <td>-1</td> <td>0</td> <td>1</td> </tr> </tbody> </table>  | $x$ | -2 | -1 | 0   | 1 | 2  | 3   | $y$ | -4 | -3 | -2 | -1 | 0                                       | 1   | <b>Rule A</b><br>$y$ is the same as $x$ multiplied by $x$ |
| $x$                | -2                               | -1  | 0   | 1  | 2  | 3   |   |    |     |     |    |    |    |    |   |     |   |
| $y$                | -4                               | -3  | -2  | -1 | 0  | 1   |   |    |     |     |    |    |    |    |   |     |   |
| <b>Graph B</b><br> | <b>Equation B</b><br>$y^2 = x$   | <b>Table B</b><br><table border="1"> <tbody> <tr> <td><math>x</math></td> <td>-2</td> <td>-1</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> </tr> <tr> <td><math>y</math></td> <td>4</td> <td>1</td> <td>0</td> <td>1</td> <td>4</td> <td>9</td> </tr> </tbody> </table>      | $x$ | -2 | -1 | 0   | 1 | 2  | 3   | $y$ | 4  | 1  | 0  | 1  | 4                                       | 9   | <b>Rule B</b><br>$x$ multiplied by $y$ is equal to 2      |
| $x$                | -2                               | -1  | 0   | 1  | 2  | 3   |   |    |     |     |    |    |    |    |   |     |   |
| $y$                | 4                                | 1   | 0   | 1  | 4  | 9   |   |    |     |     |    |    |    |    |   |     |   |
| <b>Graph C</b><br> | <b>Equation C</b><br>$y = x^2$   | <b>Table C</b><br><table border="1"> <tbody> <tr> <td><math>x</math></td> <td>0</td> <td>1</td> <td>4</td> <td>9</td> <td>16</td> </tr> <tr> <td><math>y</math></td> <td>0</td> <td>±1</td> <td>±2</td> <td>±3</td> <td>±4</td> </tr> </tbody> </table>                         | $x$ | 0  | 1  | 4   | 9 | 16 | $y$ | 0   | ±1 | ±2 | ±3 | ±4 | <b>Rule C</b><br>$y$ is 2 less than $x$ |     |   |
| $x$                | 0                                | 1   | 4   | 9  | 16 |     |   |    |     |     |    |    |    |    |   |     |   |
| $y$                | 0                                | ±1  | ±2  | ±3 | ±4 |     |   |    |     |     |    |    |    |    |   |     |   |
| <b>Graph D</b><br> | <b>Equation D</b><br>$y = x - 2$ | <b>Table D</b><br><table border="1"> <tbody> <tr> <td><math>x</math></td> <td>-2</td> <td>-1</td> <td>0</td> <td>1</td> <td>2</td> <td>4</td> </tr> <tr> <td><math>y</math></td> <td>-1</td> <td>-2</td> <td>±∞</td> <td>2</td> <td>1</td> <td>0.5</td> </tr> </tbody> </table> | $x$ | -2 | -1 | 0   | 1 | 2  | 4   | $y$ | -1 | -2 | ±∞ | 2  | 1                                       | 0.5 | <b>Rule D</b><br>$x$ is the same as $y$ multiplied by $y$ |
| $x$                | -2                               | -1  | 0   | 1  | 2  | 4   |   |    |     |     |    |    |    |    |   |     |   |
| $y$                | -1                               | -2  | ±∞  | 2  | 1  | 0.5 |   |    |     |     |    |    |    |    |   |     |   |

# Algebra I Sample Task

## Best Buy Tickets

Susie is organizing the printing of tickets for a show her friends are producing. She has collected prices from several printers and these two seem to be the best.

**SURE PRINT**  
Ticket printing  
25 tickets for \$2

**BEST PRINT**  
Tickets printed  
\$10 setting up  
plus  
\$1 for 25 tickets

Susie wants to go for the best buy

She doesn't yet know how many people are going to come.

Show Susie a couple of ways in which she could make the right decision, whatever the number.

Illustrate your advice with a couple of examples.

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# Algebra I Sample Task

## Best Buy Tickets (continued)

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# Appendix

## Resources to Support the Learning Expected by the CCSS-M

### LESSON ANALYSIS AND MODIFICATION TOOL

This rubric is endorsed by the Illinois State Board of Education (ISBE).

<http://engageny.org/resource/tri-state-quality-review-rubric-and-rating-process/> :: The Tri-State Collaborative (composed of educational leaders from Massachusetts, New York, and Rhode Island and facilitated by Achieve) has developed criterion-based rubrics and review processes to evaluate the quality of lessons and units intended to address the Common Core State Standards for Mathematics and ELA/Literacy.

### SITES FOR SAMPLE TASKS THAT SUPPORT THE LEARNING EXPECTED BY THE CCSS-M

<http://illustrativemathematics.org> :: Site run by William McCallum with the goal of having a task to illustrate every CCSS-M Standard.

<http://www.map.mathshell.org.uk> :: Mathematics Assessment Project site, with MARS tasks associated with different content and practice standards.

<http://www.turnonccmath.com/> :: Site designed to elaborate the "scientific basis" of learning trajectories research and links to the Common Core State Standards. Unpacked descriptors describe students' movement from naive to sophisticated ideas. Identifies: bridging standards; underlying cognitive principles; student misconceptions; strategies and inscriptions; and models, related representations and contexts. Also, each learning trajectory includes a diagram that gives a structural overview of the standards and descriptors within.

### PERFORMANCE TASKS THAT INFORM INSTRUCTION: A GUIDE

Source: <http://www.noycefdn.org/documents/math/howperformancetasks.pdf>; accessed 23 April, 2012

#### How Performance Tasks are Used to Inform Instruction

Performance assessment opportunities are scheduled periodically throughout the school year to provide formative information to guide instruction. Most often performance tasks are administered the first week that school is in session, a second time at the end of the first semester, a third time at the end of the third quarter and then at the end of the year. The tasks are carefully selected to measure student growth from a pre-determined perspective. The most common perspectives are listed as follows:

1. Identify a big mathematical idea linked to a standard at a grade level. The students are assessed as to whether they understand and utilize that mathematical idea with tasks throughout the year.
2. Some examples might be multiplication at third grade, rational numbers at fifth grade, proportional reasoning at seventh grade or exponential growth at ninth grade.
3. Select specific tasks that measure the learning of students after a specific unit of instruction. These would be selected according to the mathematics of the curriculum taught each quarter. A unit of instruction might involve spatial visualization. The task administered following the teaching of that unit would involve spatial visualization and be tied to the geometric standard on spatial visualization at that grade span.

4. Select different types of tasks that would measure students' problem solving abilities with non routine, unrelated problems. A set of tasks that focus on different math strands as well as elicit different types of mathematical thinking and analysis are selected. Comparing the success of students in attacking, analyzing, solving and communicating their results as the year progresses is informative.
5. Select a specific strand or mathematical idea that is taught in more depth as a student proceeds through the grades. Mathematically related tasks, appropriate to a grade level, are administered to students at three or four grade levels to see comparison over a vertical slice.
6. The same mathematical task is given to two or three grades in a grade span to assess growth as students proceed through school and become more sophisticated mathematicians. These assessments can chart and compare depth in mathematical understanding.







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